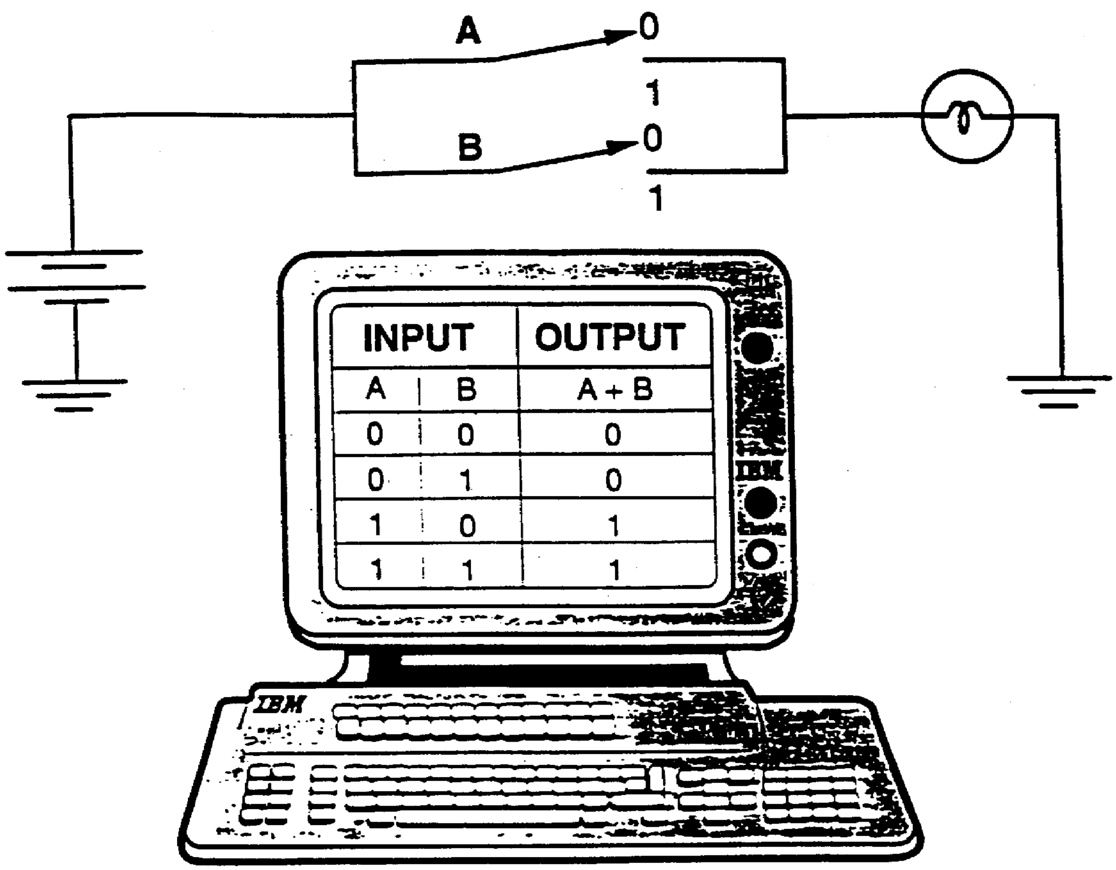
SUBCOURSE
IT 0346

EDITION
A

US ARMY INTELLIGENCE CENTER

BOOLEAN APPLICATION



THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM

AIPD

READINESS/ PROFESSIONALISM THRU GROWTH

BOOLEAN APPLICATION

Subcourse Number IT 0346

EDITION A

U.S. ARMY INTELLIGENCE CENTER
FORT HUACHUCA, AZ 85613-6000

5 Credit Hours

Edition Date: March 1997

SUBCOURSE OVERVIEW

This subcourse is designed to teach the application of Boolean Algebra. You will use skills and knowledge taught in IT 0342, IT 0343, IT 0344, and IT 0345. If you notice any difficulty during this course, review the preceding subcourses before continuing.

Subcourses IT 0342, IT 0343, IT 0344, and IT 0345 are prerequisites for this subcourse.

This lesson replaces SA 0716.

TERMINAL LEARNING OBJECTIVE

ACTION: Select the indicated function when a switch is closed, select the Boolean function of presented drawings, simplify Boolean expressions using the laws of Boolean Algebra, select the number of possible truth combinations for a given Boolean expression, select a correctly completed truth table, identify equivalent expressions, derive a minterm expression from the sum-output column of a truth table, select the minterm expression derived from the carry-output column of a truth table, choose the simplified Boolean expression derived from a Veitch diagram, select the expression equal to given logic diagrams, and complete a statement indicating the first step involved in designing logic circuits.

CONDITION: Given a diagram of a switch, a Boolean expression with up to eight variables, Veitch diagrams, logic drawings, truth tables, or written statements.

TABLE OF CONTENTS

LESSON

BOOLEAN APPLICATION

OVERVIEW

LESSON DESCRIPTION:

This subcourse is designed to teach the application of Boolean Algebra. You will use the skills and knowledge taught in IT0342, IT 0343, IT 0344, and IT 0345. If you notice any difficulty during this course, review the preceding subcourses before continuing.

TERMINAL LEARNING OBJECTIVE

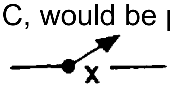
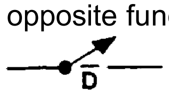
ACTION: Select the indicated function when a switch is closed, select the Boolean function of presented drawings, simplify Boolean expressions using the laws of Boolean Algebra, select the number of possible truth combinations for a given Boolean expression, select a correctly completed truth table, identify equivalent expressions, derive a minterm expression from the sum-output column of a truth table, select the minterm expression derived from the carry-output column of a truth table, choose the simplified Boolean expression derived from a Veitch diagram, select the expression equal to given logic diagrams, and complete a statement indicating the first step involved in designing logic circuits.

CONDITION: Given a diagram of a switch, a Boolean expression with up to eight variables, Veitch diagrams, logic drawings, truth tables, or written statements.

1. A digital computer uses bistable circuits as electronic switches. These electronic switches are put in a definite order to perform certain prescribed tasks, such as adding, comparing, etc. When designing switching circuits for the first digital computers, it became evident that a simple mathematical way of representing switching circuits was needed so that these circuits could be simplified. It was found that Boolean algebra could be used to represent and to simplify these electronic switching circuits.

Boolean algebra is used to ___________________ and to ___________________ electronic switching circuits.

2. Since Boolean algebra is based upon elements having two possible conditions, or states, it is readily adaptable to electronic switching circuits. A switching circuit can be in only one of two possible conditions at any given instant: it is either open, or it is closed. Using Boolean notation, a switch which may be open or closed is represented by a variable, such as A, $\overline{A}$, B, $\overline{B}$, L, $\overline{L}$, etc. To avoid confusion in this program, the labeling of a switch will indicate the function of the switch when <u>closed</u>.

1. represent simplify	2. (Continued) For example:  indicates function A is present when the switch is closed.  indicates function ______________________ is present when the switch is _______________.
	3. Select two reasons for using Boolean algebra with digital-computer electronic switching circuits. a. To simplify electronic switching circuits. b. To perform more complex mathematical operations. c. To create more complex switching circuits. d. To decrease the ease of designing electronic switching circuits. e. To represent electronic switching circuits.
	4. A switch can be in only one of two possible states at any given instant: either open or closed. If variable C is used to indicate the function present when a given switch is closed, the opposite function, $\overline{C}$, would be present when that switch is open. For example:  indicates function X is present when the switch is closed; when the switch is open, the opposite function, $\overline{X}$, is present.  indicates function _______________ is present when the switch is closed; when the switch is open, function _______________ is present.

2. $\overline{B}$

closed

5. What function is present when each of the following switches is closed?

a. b. c. d. e. f.

3. a.

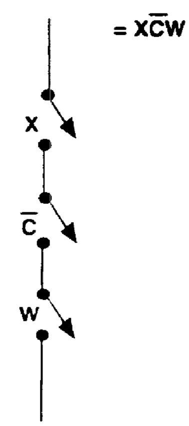
e.

6. Several fundamental switching networks are shown below with their appropriate Boolean notations. The variable (A, $\overline{B}$, C --etc.) representing each switch indicates the function is present when that switch is closed. Assuming each switching network is part of a complete circuit, the Boolean notation is as follows:

4. $\overline{D}$

a.

f.

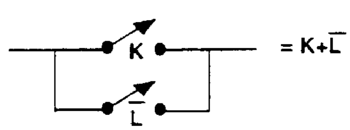


D

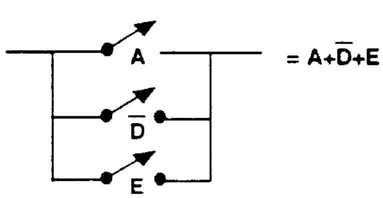
b.

c.

d.



e.



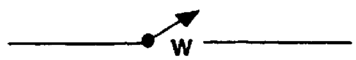
5. a. P
   b. $\bar{X}$
   c. $\bar{U}$
   d. 9
   e. D
   f. $\bar{5}$

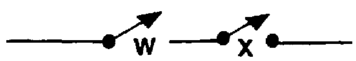
6. (Continued)

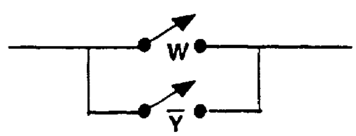
In example c, the result of switching function $A\bar{B}$ will allow current to flow only when both A and $\bar{B}$ are closed. In example d, the result of switching function $K + \bar{L}$ will allow current flow when either K or $\bar{L}$ is closed.

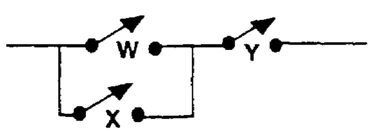
Match the switching networks in column A with the appropriate Boolean notation in column B.

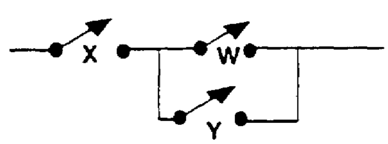
A

(1) 

(2) 

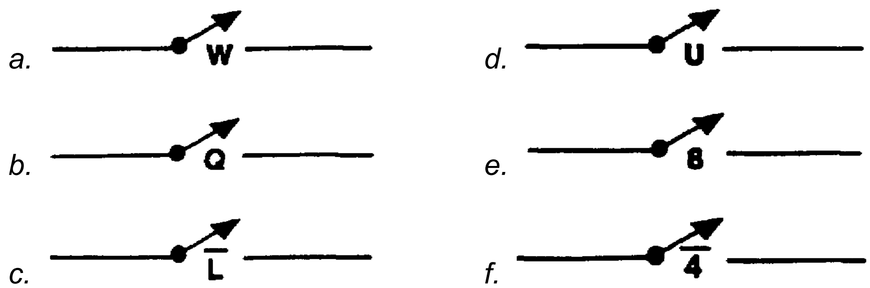
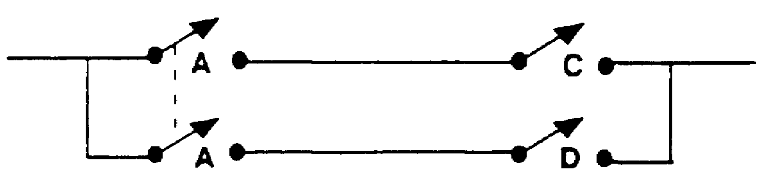
(3) 

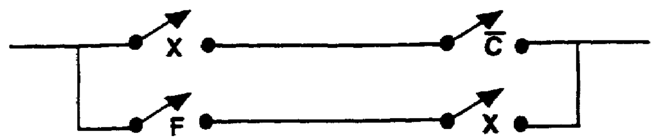
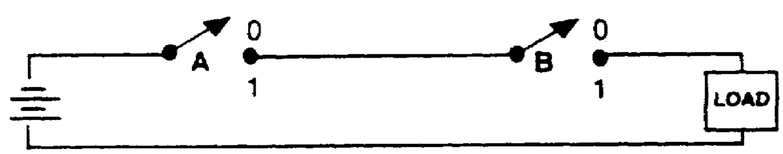
(4) 

(5) 

B

a. (W+X) Y
b. $W+\bar{Y}$
c. WX
d. W
e. X(W+Y)
f. (W+X)(W+Y)
g. W+X

6. (1) d. (2) c. (3) b. (4) a. (5) e.	7. What function is present when each of the following switches is open? 
	8. State the two reasons for using Boolean algebra with digital-computer electronic switching circuits. (1) (2)
	9. A switching network may contain a number of switches represented by the same variable. When the function of a variable is present, the function is present for every switch represented by that same variable. For example:  The switching network above contains two switches represented by variable A. When function A is present, both switches represented by variable A will be closed.

7. a. $\overline{W}$ b. $\overline{Q}$ c. L d, $\overline{U}$ e. $\overline{8}$ f. 4	9. (Continued) A switching network which contains four switches represented by variable $\overline{C}$ will have __________________ switches closed when function $\overline{C}$ is present.
8. To represent electronic switching circuits. To simplify electronic switching circuits.	10. The figure below represents a switching network which contains four individual switches. Notice that two of the switches are represented by the same variable, X.  When the function of variable X is present for this network, __________________________ represented by variable X will (both switches/only one switch) be ______________. (closed/open)
	11. A Boolean expression can be written for switching circuits after first determining which combination of switches must be caused to obtain an output. For example, the AND circuit of two switches (variables) is shown below. 

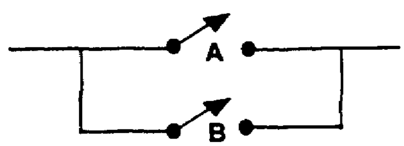
9. four

10. both switches closed

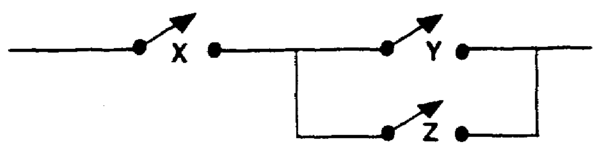
11. (Continued)

Switch A or switch B may be either open or closed (O or 1 position). The series circuit will provide an output only if both A and B are closed; i.e., equal to 1. If either switch A or switch B is open (i.e., equal to 0), then the circuit will not provide an output. The Boolean expression representing this switching circuit is AB.

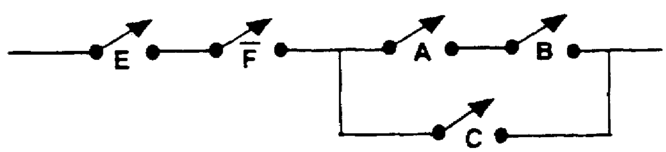
Assuming there is a complete circuit for each of the following switching circuits, the Boolean expressions representing the output would be



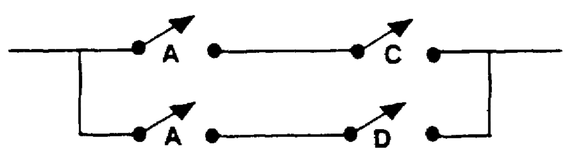
$= A + B$



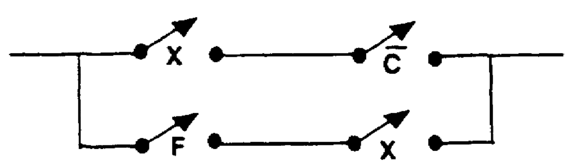
$= X(Y + Z)$



$= E\overline{F}(AB + C)$

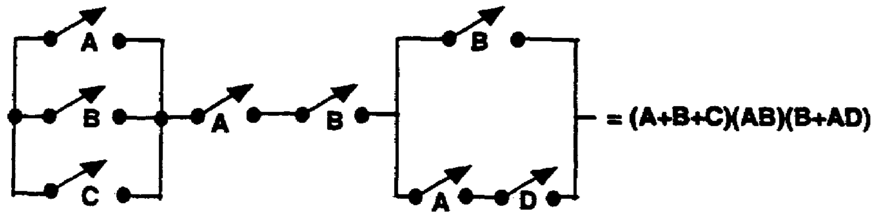


$= AC + AD$



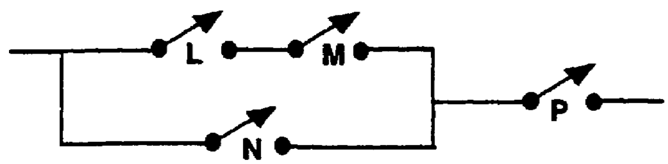
$= X\overline{C} + FX$

11. (Continued)

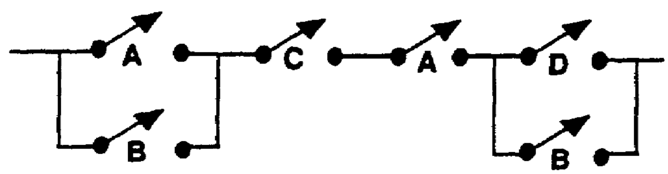


Write Boolean expressions for the following switching circuits:

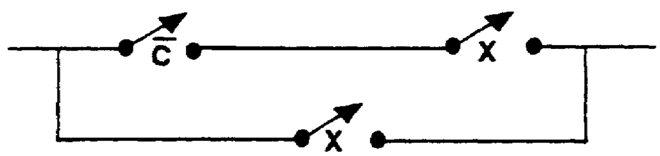
a.



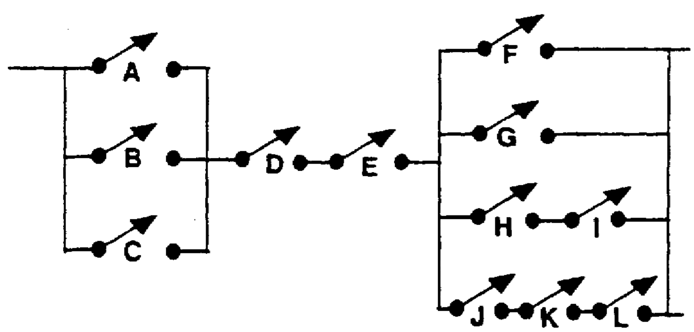
b.



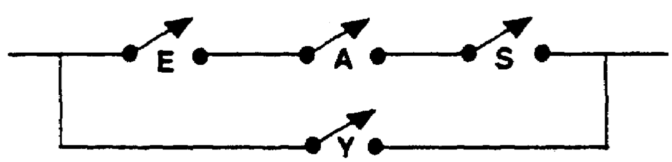
c.



d.



e.



11.

a. (LM+N) P

b. (A+B)(CA)
(D+B)

c. $\overline{C}$X+X

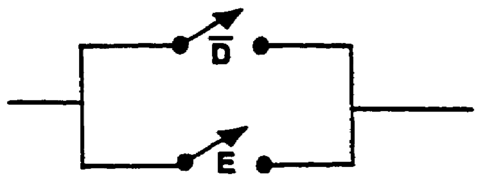
d. (A+B+C)(DE)
(F+G+HI+JKL)

e. EAS+Y

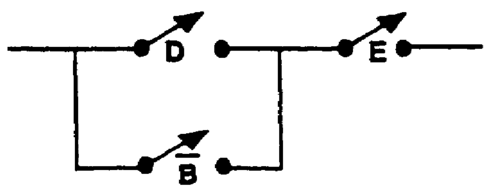
12. Match each of the switching networks in column A with the appropriate Boolean expression in column B.

A

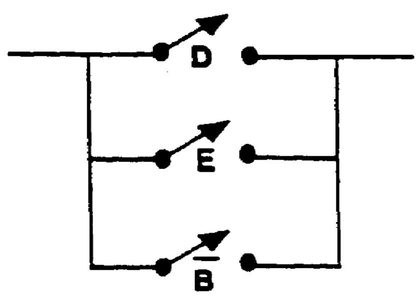
(1) 



(2)



(3)



(4)

B

a. (D+$\overline{B}$) E

b. D+D+$\overline{E}$

c. D+E

d. $\overline{D}$E

13. TEST FRAME
What function is present when each of the following switches is closed?

a. 

e. 

b. 

f. 

c. 

g. 

d. 

12.

(1) d.

(2) c.

(3) a.

(4) b.

13.

a. B

b. O

c. $\overline{O}$

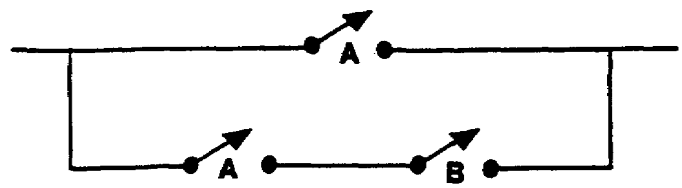
d. $\overline{L}$

e. E

f. A

g. $\overline{N}$

14. One of the applications of Boolean algebra is the simplification of switching networks. For example, the switching network shown below can be simplified to a circuit consisting of only one switch which performs the same function.



The switching network shown above is simplified as follows:

Step 1. Write the Boolean expression for the network.

A + AB

Step 2. Simplify the expression, using the basic laws of Boolean algebra.

A + AB = original expression

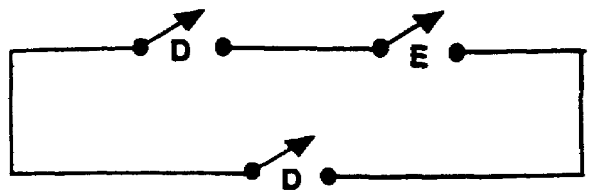
A = result of absorption (A + AB = A)

Step 3. Draw the switching network which represents the simplified expression.

14. (Continued)

The original switching network, consisting of three switches, has been simplified to one switch which performs the same function.

Simplify the switching network below, using the basic laws of Boolean algebra. Draw the simplified circuit.



SOLUTION TO FRAME 14:

Step 1. D +DE

Step 2. D + DE = original expression

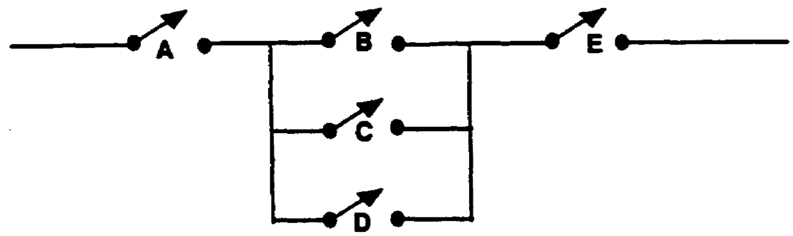
D = result of absorption (D + DE = D)

D = simplified expression

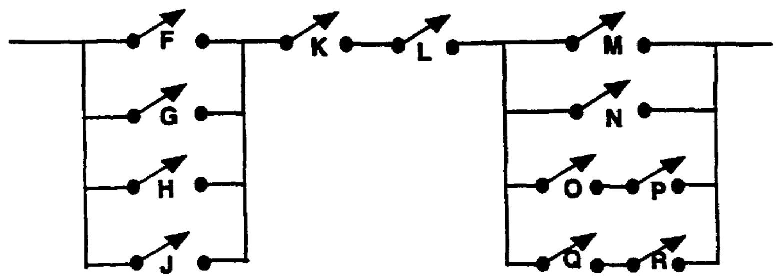
Step 3.

15. Write the Boolean expressions for the following switching circuits:

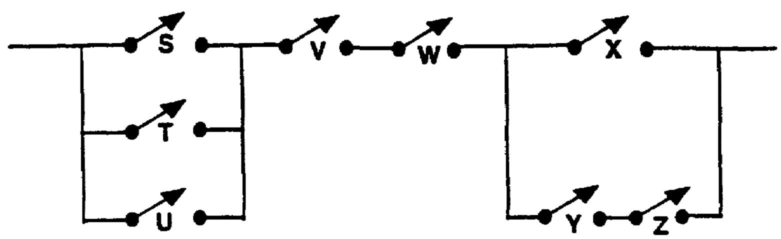
a.



b.

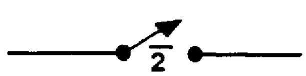


c.



16. What function is present when each of the following switches is open?

a.



d.

b.

e.

c.

f.

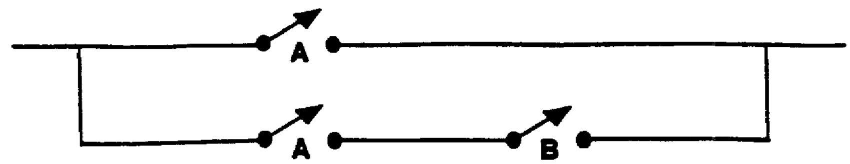
15.

a. A(B+C+D) E

b. (F+G+H+J)
(KL)(M+N
+OP+QR)

c. (S+T+U)(VW)
(X+YZ)

17. Simplify the switching circuit below, using the basic laws of Boolean algebra. Draw the simplified circuit.



16.

a. 2

b. $\overline{G}$

c. F

d. $\overline{6}$

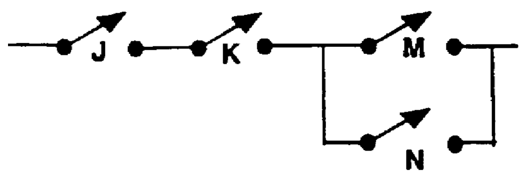
e. $\overline{K}$

f. E

18. Match each of the switching networks in column A with the appropriate Boolean expression in column B.

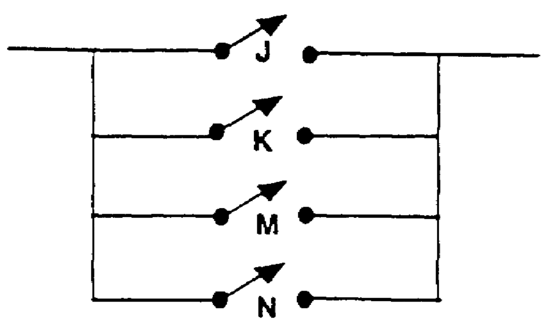
A

A



(1)

(2)



(3)

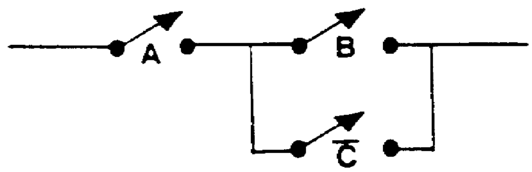
B

a. ABC

b. $A(B+\overline{C})$

c. JK(M+N)

d. J+K+M+N



(4)

17.

18.

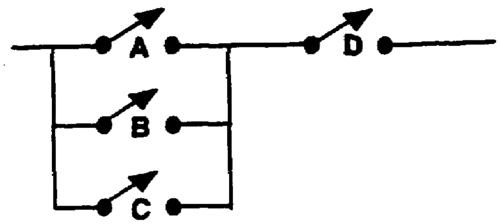
(1) c.

(2) a.

(3) d.

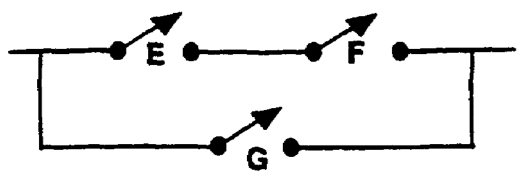
(4) b.

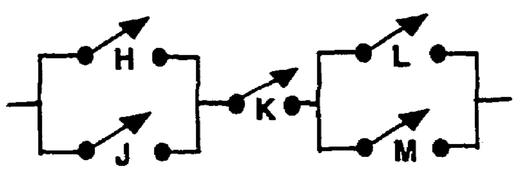
19. Write the Boolean expressions for the following switching circuits:



a.

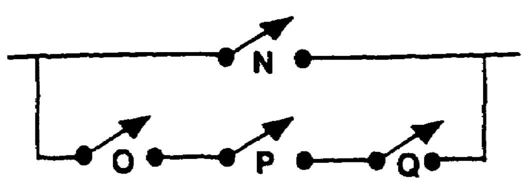
b.





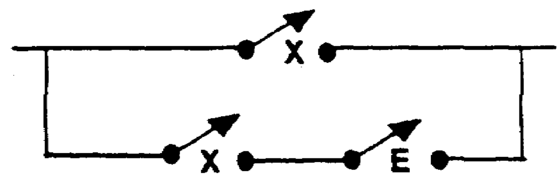
c.

d.



20. Simplify the switching circuit below, using the basic law of

Boolean algebra. Draw the simplified circuit.



19.

a. (A+B+C) D

b. (EF) +G

c. (H+J) K(L+M)

d. N+(OPQ)

21. The objective of using Boolean algebra in digital-computer study is to determine the 'truth value' of a combination of two or more statements. For any Boolean function, there is a corresponding truth table, which shows in tabulated form the true condition of the function for each occasion in which conditions can be assigned to its variables. In binary Boolean algebra, 0 and 1 are the symbols assigned to the variables of any function.

The objective of using Boolean algebra in digital-computer study is to determine the ______________________________ of the combination of two or more statements.

20

22. In designing logic circuits for a computer, the first step is to construct a truth table. The truth table not only provides a ready reference for use in analyzing the operating theory of the circuit, but also is useful in developing the overall signal-flow diagram.

The first step in designing logic circuits for computers is to construct a ___________________________________.

21. truth value	23. The objective of using Boolean algebra in digital-computer study is to determine the ________________________ of the combination of ____________ or ____________ statements.
22. truth table	24. The number of possible truth combinations of a given number (n) of binary variables is 2^n. 2 binary variables = $2^n = 2^2 = 4$ combinations 3 binary variables = $2^n = 2^3 = 8$ combinations 4 binary variables = $2^n = 2^4 = 16$ combinations To construct a truth table for the Boolean expression AB will require 2^2, or 4, rows--one row for each truth combination, as indicated below.

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

How many possible truth combinations are there for the following Boolean expressions?

a. DE

b. NREV

c. L+A

d. N+A+D

e. EZ+RA

f. NAVY

23. truth value two more	25. The first step in designing logic circuits for computers is to ____________________ a ____________________ ______________________.
24. a. $2^2 = 4$ b. $2^4 = 16$ c. $2^2 = 4$ d. $2^3 = 8$ e. $2^4 = 16$ f. $2^4 = 16$	26. Write the number of possible truth combinations for the following Boolean expressions: a. (A+B+C) D b. (EF) +G c. N+OP+QR d. U+V
	27. What is the objective of using Boolean algebra in digital-computer study?
	28. What is the first step when designing logic circuits for digital computers?

25. construct truth table	29. Write the number of possible truth combinations for the following Boolean expressions: a. SO+ON b. W+E c. WILL d. B+E e. D+ONE
26. a. $2^4 = 16$ b. $2^3 = 8$ c. $2^5 = 32$ d. $2^2 = 4$	30. Assume a problem is presented that requires the design of a circuit which will add two binary digits. The first step is to construct a truth table which includes an output for each combination of possible input states. From the rules for binary addition, a truth table is derived as shown below.
27. To determine the truth value of the combination of two or more statements.	
28. construct truth table	

0 + 0 = 0 with a carry of 0
0 + 1 = 1 with a carry of 0
1 + 0 = 1 with a carry of 0
1 + 1 = 0 with a carry of 1

Rules for binary addition

INPUTS		**OUTPUT**	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Truth table derived from the rules for binary addition

Construct a truth table for a circuit that will add two binary digits. Label the inputs C and D. Show the SUM and the CARRY outputs.

29.

a. 2^3=8

b. 2^2=4

c. $2^3 = 8$

d. $2^2 = 4$

e. 2^4 =16

30

INPUTS		OUTPUTS	
C	D	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

31. The truth table provides a simple means for testing the logical equivalence of two or more Boolean expressions. To test the logical equivalence of two expressions, construct a truth table for both expressions. If both expressions have the same truth value for each case (row) in the truth table, the expressions are equivalent and can be substituted for each other. For example: Prove that A+AB=A.

A	B	AB	A+AB	A+AB=A
0	0	0	0	1
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

The third column in the truth table above represents the logical product (AB) for two variables. The fourth column represents the logical product and sum (A+AB) of two variables. A "1" is placed in this column when either the A column or the AB column or both have a truth value of 1. The fifth column is a comparison of the first column (A) and the fourth column (A+AB). A 1 is placed in this column whenever the truth value of the two columns is the same; i.e., both 0 or both 1.

31. (Continued)

Since the final column results in all 1's, the expression

A+AB = A is proved equivalent in all cases.

a. Complete the following truth table:

A	**B**	**A+B**	**A(A+B)**	**A(A+B) =A**
0	0	0	0	
0	1	1	0	
1	0	1	1	
1	1	1	1	

b. The expression A(A+B) = A _______________
(is/is not)
equivalent.

32. Construct a truth table which will add two binary digits. Label the inputs E and F. Show both the SUM and the CARRY outputs.

INPUTS		**OUTPUTS**	
		SUM	CARRY

31.
a. A(A+B) = A
1
1
1

b. is

32

E	F	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

33. For a function to be performed, a MINTERM EXPRESSION can be derived for each output column from a truth table. One output column is taken at a time, and a Boolean product is written for each output condition that is a 1. After the Boolean product which represents each 1 output is written, the outputs are ORed together to form a MINTERM EXPRESSION. For example, the diagram below is the truth table for a circuit that will add two binary digits.

INPUTS		**OUTPUT**	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

To derive a MINTERM EXPRESSION for each output column, the Boolean product is written for each 1 output. Use only one output column at a time. The Boolean product for the SUM-output column is derived as follows:

The first 1 occurs when A is 0 and B is 1; thus, the Boolean product for this condition is AB.

The next 1 occurs when A is 1 and B is 0; thus, the Boolean product for this condition is AB.

33. (Continued)

Either Boolean product results in a 1 output. The sum-output column MINTERM EXPRESSION becomes AB+AB by combining the two Boolean products using the OR operation. Derive a minterm expression for the 1 output in the CARRY-output column of the truth table below.

INPUTS		OUTPUT	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

34. Complete the truth table below.

a.

X	Y	X+Y	X(X+Y)	X(X+Y) =X
0	0	0	0	
0	1	1	0	
1	0	1	1	
1	1	1	1	

b. Is the expression X(X+Y) =X equivalent in all cases?

33. AB

<u>Solution</u>:

A 1 output occurs in the CARRY column when A is 1 and B is 1; thus, the Boolean product for this condition is AB.

34.

a. X(X+Y) =X

1
1
1
1

b. Yes.

35. The truth table below represents a circuit which will be encountered in digital computers. Notice, there are three binary variables, A, B, and K. The possible truth combinations are $2^n = 2^3 = 8$, as shown.

A	B	K	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

To derive a minterm expression for the SUM-output column, first write the Boolean product for each output which is a 1 in the SUM column. After the Boolean product is written for each 1 output, the outputs are ORed together to form a minterm expression. The SUM-output minterm expression derived from the truth table above is

$$\bar{A}\bar{B}K + \bar{A}B\bar{K} + A\bar{B}\bar{K} + ABK$$

Derive the minterm expression from the CARRY output of the truth table above.

35. $\overline{A}BK + A\overline{B}K$

$+ AB\overline{K} + ABK$

36. Derive a minterm expression from the SUM-output column of the truth table below.

INPUTS		**OUTPUTS**	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

37. Construct a truth table for adding two binary digits. Label the inputs A and B. Show both the SUM and the CARRY outputs.

INPUTS		OUTPUTS	

36. AB+AB

37

E	F	SUM	CARRY

38. Minterm expressions which have been derived from a truth table may not always be in the simplest form. If the minterm expressions are plotted on a Veitch diagram, a simplified expression may be extracted. The SUM and the CARRY output minterm expressions are plotted on the Veitch diagrams below.

$SUM=\bar{A}\bar{B}K+\bar{A}B\bar{K}+A\bar{B}\bar{K}+ABK$

	A		$\bar{A}$	
	$\bar{K}$	K	K	$\bar{K}$
B		X		X
$\bar{B}$	X		X	

$CARRY=\bar{A}BK+A\bar{B}K+AB\bar{K}+ABK$

	A		$\bar{A}$	
	$\bar{K}$	K	K	$\bar{K}$
B	X	X	X	
$\bar{B}$		X		

Investigation reveals the expression extracted from the SUM Veitch diagram is exactly the same as the expression is in the simplest form.

Extract the simplified expression from the Veitch diagram of the CARRY function.

38. AB + BK +AK

39. Derive a minterm expression from the SUM-output column of the truth table below.

INPUTS			OUTPUT	
A	B	K	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

40. Complete the truth table below.

a.

X	Y	X+Y	X(X+Y)	X(X+Y) = X
0	0	0	0	
0	1	1	0	
1	0	1	1	
1	1	1	1	

b. Is the expression X(X+Y) =X equivalent in all cases?

39. $\bar{A}\bar{B}K + \bar{A}B\bar{K}$

$+ A\bar{B}\bar{K} + ABK$

40. a. X(X+Y) =X

1
1
1

b. Yes

41. After a simplified expression has been extracted from a Veitch diagram, the final step in designing logic circuits, using Boolean algebra, is to draw the appropriate logic diagram to represent the simplified expression. The sum expression and the carry expression for a circuit which will add two binary digits are

$\bar{A}B + A\bar{B}$ SUM EXPRESSION

AB CARRY EXPRESSION

Since there are two outputs consisting of (1) the sum output and (2) the carry output, the logic diagram must show two outputs when completed.

To diagram the sum output, first recognize the expression $\bar{A}B + A\bar{B}$ as an overall two-input OR circuit containing inputs $\bar{A}B$ and $A\bar{B}$. The first step in drawing the logic diagram is shown below.

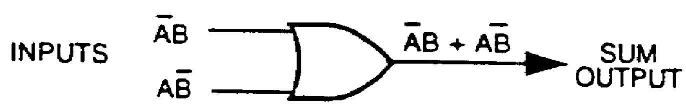
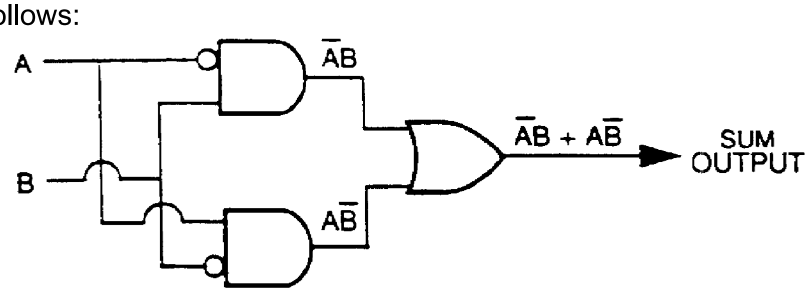
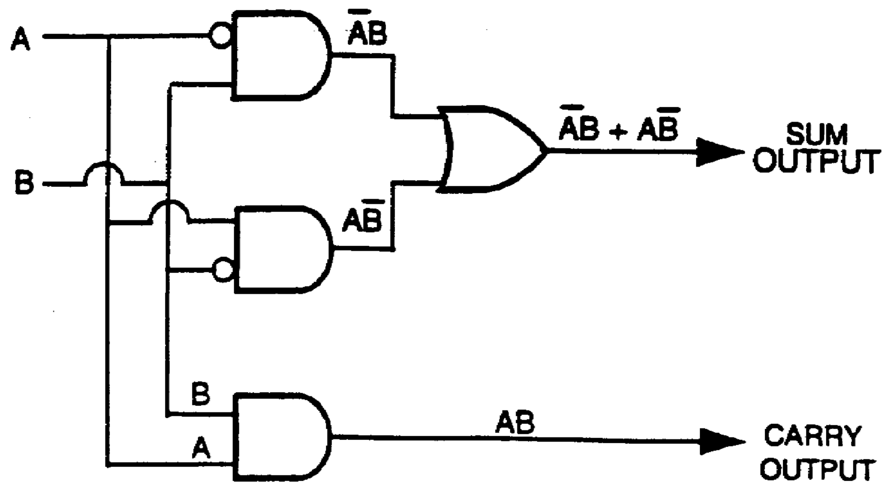


Diagram each input and draw the appropriate logic symbols as follows:



41. (Continued)

To diagram the carry function (AB) of the circuit, simply tie inputs A and B into an AND-logic symbol and indicate the carry function as follows:

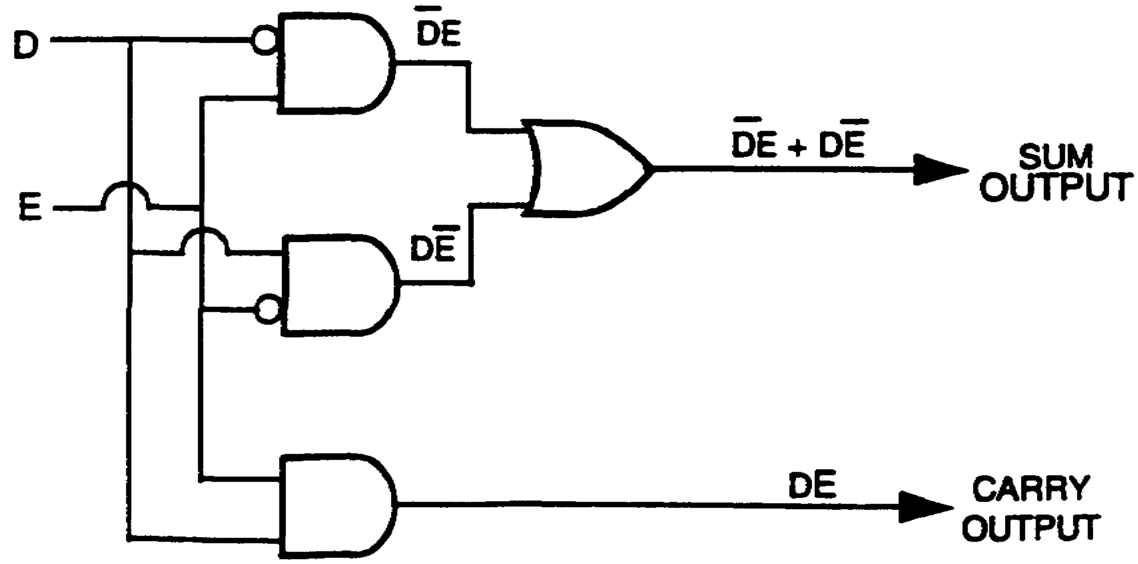


Draw the logic diagram for the following expressions which describe a circuit that will add two binary digits. Label the two outputs.

$\bar{D}E + \bar{D}E$ = SUM OUTPUT

DE = CARRY OUTPUT

SOLUTION TO FRAME 41:



42. Extract a simplified Boolean expression from the Veitch diagram below.

	R	R	$\bar{R}$	$\bar{R}$
T			X	X
$\bar{T}$		X		
	$\bar{Z}$	Z	Z	$\bar{Z}$

43. Derive a minterm expression from the SUM-output column of the truth table below.

A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

42. $R\overline{T}Z+\overline{R}T$

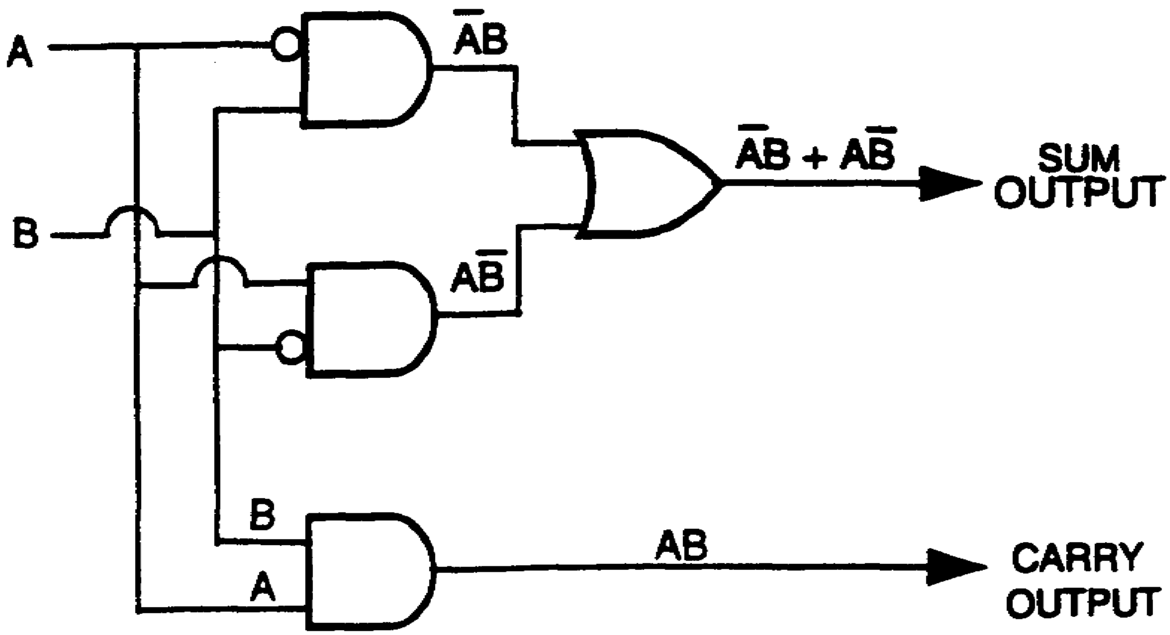
43. $\overline{A}B+A\overline{B}$

44. The Boolean expressions below describe a circuit which will add two binary digits. Draw the logic diagram for the circuit and label each output.

$\overline{A}B + A\overline{B}$ = SUM

AB = CARRY

SOLUTION TO FRAME 44:



45. Derive a minterm expression from the CARRY-output column of the truth table below.

INPUTS			**OUTPUT**	
A	B	K	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

46. Extract a simplified Boolean expression from the Veitch diagram below.

	D		$\bar{D}$	
F	X		X	
$\bar{F}$			X	
	$\bar{J}$	J	J	$\bar{J}$

45. $\bar{A}BK + A\bar{B}K$
$+ AB\bar{K} + ABK$

46. $DF\bar{J} + \bar{D}J$

47. The Boolean expressions below describe a circuit which will add two binary digits. Draw the logic diagram for the circuit and label each output.

$\bar{C}L + C\bar{L}$ = SUM

CL = CARRY

SOLUTION TO FRAME 47: