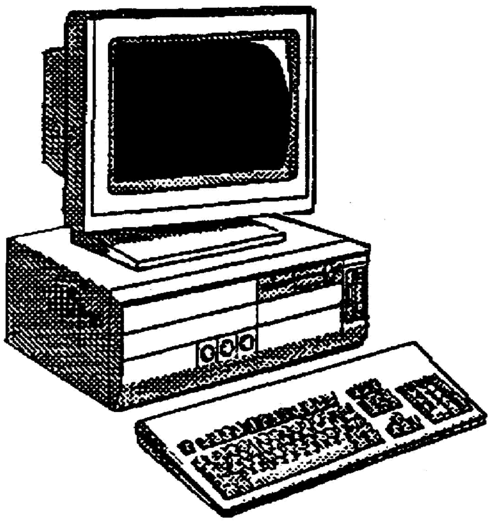
SUBCOURSE
IT 0344

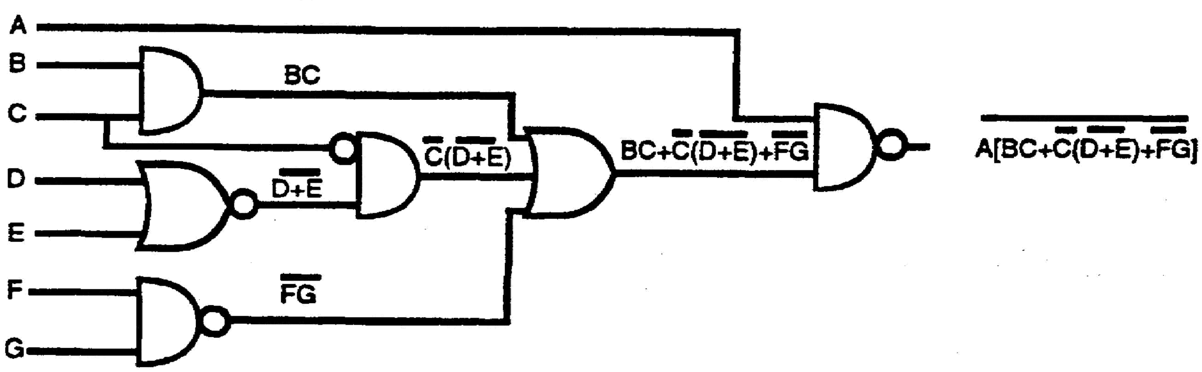
EDITION
A

US ARMY INTELLIGENCE CENTER

BASIC LAWS OF BOOLEAN ALGEBRA



BOOLEAN LAWS ?



THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM

AIPD

READINESS/ PROFESSIONALISM THRU GROWTH

BASIC LAWS OF BOOLEAN ALGEBRA

Subcourse Number IT 0344

EDITION A

US ARMY INTELLIGENCE CENTER
Fort Huachuca, AZ 85613-6000

5 Credit Hours

Edition Date: JULY 1997

SUBCOURSE OVERVIEW

This subcourse is designed to teach you the application of the basic laws of Boolean Algebra to the simplification of electronic circuits.

Prerequisites for this subcourse are Subcourses IT 0342 and IT 0343.

This subcourse replaces SA 0714.

ACTION: You will be able to simplify algebraic expressions of electronic circuits by applying the basic laws and identities of Boolean Algebra.

CONDITION: Given algebraic expressions of electronic circuits and a summary of the basic laws and identities.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70% on the Subcourse Examination.

TABLE OF CONTENTS

LESSON

BASIC LAWS OF BOOLEAN ALGEBRA

OVERVIEW

LESSON DESCRIPTION:

In this lesson you will learn to apply the basic laws of Boolean Algebra to the simplification of electronic circuits.

TERMINAL LEARNING OBJECTIVE:

ACTION: Simplify algebraic expressions of electronic circuits by applying the basic laws and identities of Boolean Algebra.

CONDITION: Given algebraic expressions of electronic circuits and a summary of the basic laws and identities.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70% on the subcourse examination.

SUBCOURSE ORGANIZATION. The pages of this subcourse are divided into two sections, the Frames and responses/answers. The frames are the largest area of the page and they are numbered sections. The responses/answers are not numbered and are on the left side of each page. The correct response/answer to a frame is in the following response/answer block. Periodically, there are additional instructions in the response/answer block referring you to other areas in the subcourse that contain step by step solutions.

	1. Boolean Algebra aids the computer technician in understanding electronic computer circuits. The basic laws of Boolean Algebra are used to manipulate and simplify Boolean expressions. The simplest electronic circuits which will accomplish a particular function are designed by applying the basic laws to manipulate and simplify Boolean expressions. Keep in mind, however, that the laws of Boolean Algebra do not correspond exactly to the laws of ordinary Algebra. Boolean Algebra is used to ______________________________ and _______________________ Boolean expressions.
manipulate simplify	2. Boolean Algebra utilizes the binary numbering system. In the binary numbering system, there are only two possible constants, '1 and 0, and only two possible values for any variable, also 1 and 0. A Boolean expression can be multiplied by following the same rules used in ordinary Algebra; however, since the largest value in the binary numbering system is 1, no squares are generated. For example, in ordinary Algebra, the expressions (A+B)(A+B) multiplied together are expressed as $A^2+AB+BA+B^2$. In Boolean Algebra, the same expressions (A+B)(A+B), multiplied together are expressed as AA+AB+BA+BB. In Boolean Algebra, A multiplied by A (A•A) is indicated as AA; B multiplied by B (B•B) is indicated as BB, etc.

	2. (Continued) Multiply the Boolean expressions below, using Boolean Algebra. a. (A+D)(A+C) c. (A+B)(AB+C) b. (AB+C)(R+S) d. (A)(A+B)
a. AA+AC+DA +DC b. ABR+ABS+CR +CS c. AAB+AC+BAB +BC d. AA+AB	3. Boolean Algebra is used to a. convert Boolean expressions to binary O's and 1's. b. communicate with digital computers in binary. c. manipulate and simplify Boolean expressions. d. simplify Boolean expressions involving complex Algebra.
c.	4. Multiply the following Boolean expressions: a. (J+R)(RT) c. P(T+P+PT) b. (LM)(M+L+T) d. (A+N)(N+C)

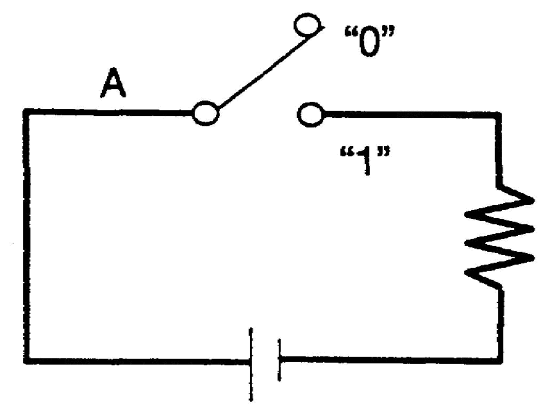
a. JRT+RRT

b. LMM+LML
+LMT

c. PT+PP+PPT

d. AN+AC+NN
+NC

5. Boolean Algebra is a method of mathematically representing logical operations. In order to represent logical operations, a set of basic laws and identities has been developed for Boolean Algebra. It is not as important to know these laws and identities by name as it is to know when and how to put them to use. One of the basic laws of Boolean Algebra is the law of IDENTITY, which states: “Any letter, number, or expression is equal to itself." For example, A = A; B = B $\overline{C} = \overline{C}$; $\overline{R}J = \overline{R}J$; etc. The switching-circuit diagram and the truth table of the law of IDENTITY are shown below.



A	A
0	0
1	1

LAW OF IDENTITY

Select examples of the law of IDENTITY from the following.

a. 1 = 1

b. A = B

c. A = 1

d. B = B

e. A = $\overline{B}$

f. $\overline{O} = \overline{O}$

g. A = $\overline{A}$

h. AB = AB

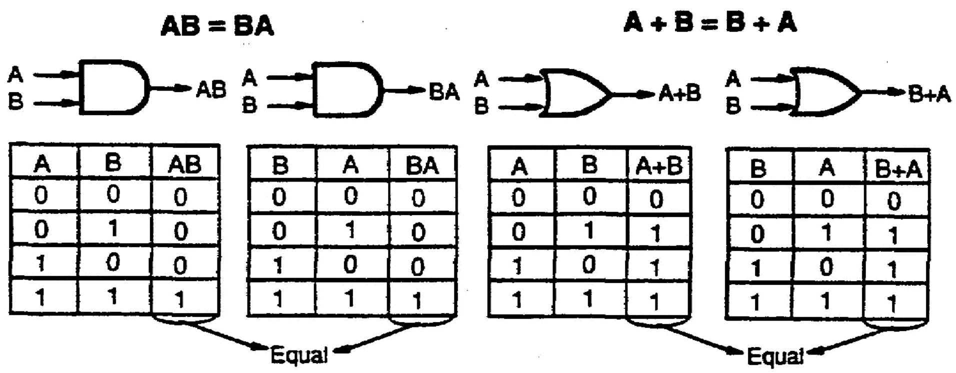
a.

d.

f.

h.

6. Another basic law of Boolean Algebra is the COMMUTATIVE law, which states: “When inputs to a logic diagram are ANDed or ORed, the order In which the output Boolean expression is written does not affect the binary value of the output." For example, AB = BA; A+B = B+A; PDQ = QPD; C+J+S = J+C+S; etc. The logic diagrams and truth tables below show that the output Boolean expression of an AND circuit or an OR circuit can be written without regard to the order of the factors.



A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

B	A	BA
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

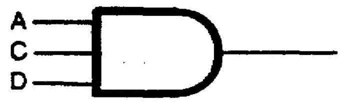
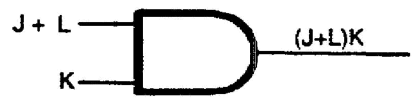
B	A	B+A
0	0	0
0	1	1
1	0	1
1	1	1

COMMUTATIVE LAW

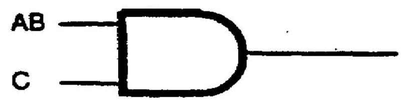
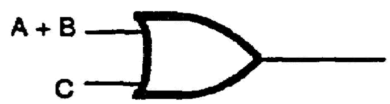
Indicate two ways the output Boolean expressions can be written for each of the following.

a.

b.

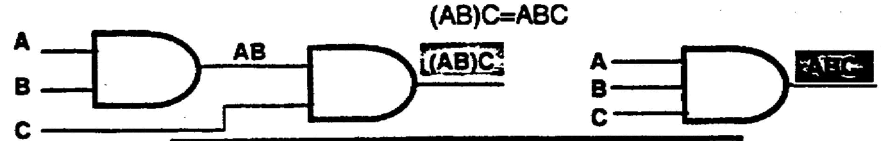
a. A+T T+A b. AQ QA	7. In the logic diagram below, the output may be written ACD. List two additional ways the output can also be correctly written. 
ADC CAD CDA DAC DCA (Any two.)	8. When applying the COMMUTATIVE law, observe signs of grouping and apply the law to only the logic diagram under consideration.   As illustrated above, the Boolean expressions (J+K) L and (J+L) K for the logic diagrams contain the same variables, but the expressions are not equal. According to the COMMUTATIVE law, is (M+N) S equal to (M+S) N?
No.	9. Select examples of the law of IDENTITY. a. $XYZ = \bar{X}Y\bar{Z}$ b. $K\bar{L}T = K\bar{L}T$ c. TUB = TUB d. ABCD = ABD e. MN= MN f. AT=TD

b. c. e.	10. Using the COMMUTATIVE law, select the expressions below which <u>are equal</u>. a. K ($\overline{L}$+$\overline{M}$) and ($\overline{M}$+L) K b. W+X+YZ and ZY+X+W c. A(BC+D+$\overline{E}$) and ($\overline{E}$+BC+D) A d. GH+J and HI+G e. (D+F) E and E(F+D)
b. c. e.	11. To aid in simplifying Boolean expressions, learn to recognize terms within the expressions which are equal. For example: BCC + ADE + DEA EQUAL Circle the terms which are equal in each expression below. a. AB +AC + BA b. CE+$\overline{A}$+$\overline{E}\overline{C}$+$\overline{C}\overline{E}$ c. EB+AG+BE+$\overline{AG}$

a. AB, BA b. $\bar{E}\bar{C}$, $\bar{C}$ $\bar{E}$ c. EB, B E	12. Two possible combinations of inputs to logic diagrams are shown below. a.  <u>AND</u>ed INPUT TO <u>AND</u> SYMBOL b.  <u>OR</u>ed INPUT TO <u>OR</u> SYMBOL Write the output Boolean expression for the diagrams above. a. b.
a. (AB) C b. (A+B) +C	13. The output Boolean expressions of the logic diagrams above contain parentheses to indicate grouping. It is possible to remove the parentheses from the output Boolean expressions, which will result in simplified diagrams. To remove the parentheses, the ASSOCIATIVE law is used. The ASSOCIATIVE law is illustrated on the following page with the corresponding logic diagrams and truth tables.

13. (Continued)

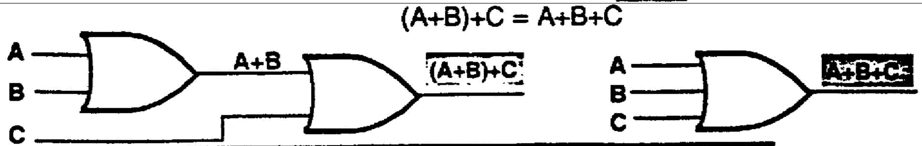
(AB)C=ABC



A	B	C	AB	(AB)C	ABC
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	0	0
1	1	1	1	1	1

EQUAL

(A+B)+C = A+B+C



A	B	C	A+B	(A+B)+C	A+B+C
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

ASSOCIATIVE LAW EQUAL

The logic diagrams and truth tables above prove that the parentheses can be removed and a simplified diagram obtained when there is an ANDed input to an AND gate or an ORed input to an OR gate. Simplify the following Boolean expressions, using the ASSOCIATIVE law.

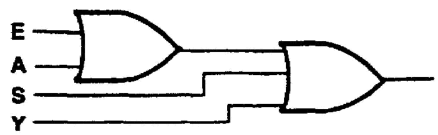
a. R (ST)

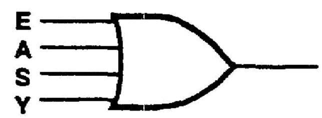
b. L+ (M+R)

c. (A+B+C) +D

d. R (US) T

<table>
<tr>
<td>a. RST
b. L+M+R
c. A+B+C+D
d. RUST</td>
<td>14. A simplified expression, which resulted in a simplified logic diagram, was obtained in frame 13 by using the ASSOCIATIVE law. The simplified logic diagram enables a circuit to be constructed which will be more economical, while still performing the desired function. Which logic diagram below will result in the simplest, most economical circuit?

a.


b.

</td>
</tr>
<tr>
<td>b.</td>
<td>15. Using the ASSOCIATIVE law, simplify the following Boolean expressions:

a. AB (CDE)

b. C+(D+E) + (F+G)

c. (UV)(WX)(YZ)

d. (T+UV) +t (W+ XYZ)</td>
</tr>
</table>

a. ABCDE

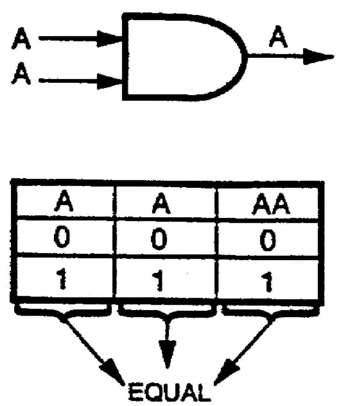
b. C+D+E+F+G

c. UVWXYZ

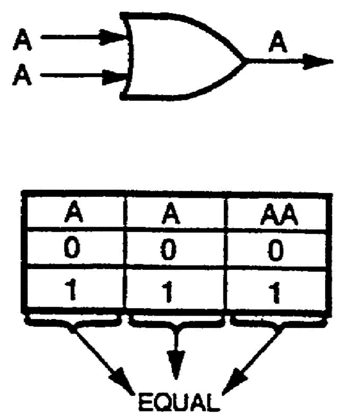
d. T+UV+W+XYZ

16.

AA=A



A + A = A

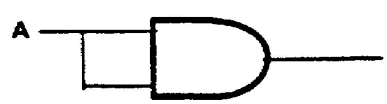


IDEMPOTENT LAW

The IDEMPOTENT law is illustrated above with the corresponding truth tables. If input A is ANDed with input A or if input A is ORed with input A, the output will have a binary value of A.

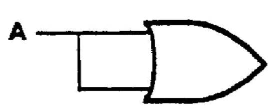
Complete the truth tables for the logic diagrams below.

a.



A	A	AA
0	0	
1	1	

b.



A	A	A+A
0	0	
1	1	

a.

A	A	AA
0	0	0
1	1	1

b.

A	A	A+A
0	0	0
1	1	1

17. Using the IDEMPOTENT law, simplify the following Boolean expressions:

a. P+P

b. A+B+A+C+B+D

c. (RS)(RS)

d. $X\bar{Y}\bar{Z}+X\bar{Y}\bar{Z}$

---

a. P

b. A+B+C+D

c. RS

d. $X\bar{Y}\bar{Z}$

18. The laws which have been covered up to this point are listed below.

IDENTITY:	A=A	$\bar{A}=\bar{A}$
COMMUTATIVE:	AB = BA	A+B = B+A
ASSOCIATIVE:	A (BC) = ABC	A+ (B+C) = A+B+C
IDEMPOTENT:	AA = A	A+A = A

These are a few of the laws which are used to simplify Boolean expressions. For example:

(XY) + (YX) + (AB) + (BA) + (EE)

COMMUTATIVE

(XY) + (XY) + (AB) + (AB) + (EE)

IDEMPOTENT

(XY) + (AB) + (E)

ASSOCIATIVE

XY+AB+E

Simplify the following Boolean expressions by using the required laws.

a. (LL)(MM)

18. (Continued)

b. FG+(FG+H)

c. SR+(RT+RS)

d. (ST+Z)($\overline{AR}$)($\overline{RA}$)(Z+TS)

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a. LM

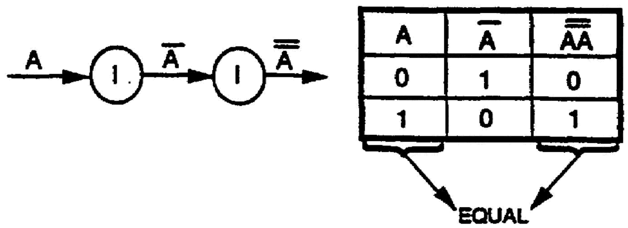
b. FG+H

c. SR+RT

d. (ST+Z)(AR)

Step by step solutions on page A-2.

19.



LAW OF DOUBLE NEGATION: $\overline{\overline{A}}$ = A

The diagram and truth table above represent the law of DOUBLE NEGATION. Whenever two vincula of equal length extend over the same variable or expression, both vincula may be removed by using the DOUBLE NEGATIVE law. For example, $\overline{\overline{A}}$ = A; $\overline{\overline{CD}}$ = CD; $\overline{\overline{A+B}}$ = A+B; etc.

Simplify the following Boolean expressions, using the DOUBLE NEGATIVE law.

a. $\overline{\overline{X}}\ \overline{\overline{Y}}\ \overline{\overline{Z}}$

b. $\overline{\overline{AB}}$+X

c. $\overline{\overline{(R+S)}}$ T

d. L $\overline{\overline{M}}\ \overline{T}$

a. XYZ b. AB+X c. (R+S) T d. $LM\overline{T}$	20. When more than two vincula of equal length extend over the same variable, term, or expression, the vincula may be removed <u>two at one time</u> by using the DOUBLE NEGATIVE law. For example, $\overline{\overline{A}} = A$; $\overline{\overline{CD}} = CD$; $\overline{\overline{\overline{X+Y}}} = \overline{X+Y}$; etc. By using the DOUBLE NEGATIVE law, _____________ vincula may be removed at the same time.
two	21. Simplify the following Boolean expressions, using the DOUBLE NEGATIVE law. a. $\overline{\overline{AB}} + C$ b. $\overline{\overline{B}} + \overline{\overline{F}} + GH$ c. $(R + S)\ T + \overline{\overline{Z}}$ d. $AB + \overline{\overline{BD}}$
a. AB+C b. B+F+GH c. (R+S) T+Z d. AB+BD	22. Simplify the following expressions, using the laws that have been covered. a. $\overline{\overline{C}} + \overline{\overline{DG}} + \overline{\overline{FD}} + G + \overline{\overline{C}}$ b. $(\overline{\overline{Q+R}} + \overline{\overline{S}}) + \overline{\overline{S}} + (R + Q)$ c. $(M\overline{\overline{NP}} + Q)\ L + \overline{\overline{K}}$ d. $(\overline{\overline{WX}}Y + Z)\ (Z + \overline{\overline{WX}}Y)$

a. $C+DG+\overline{FD}+G$.

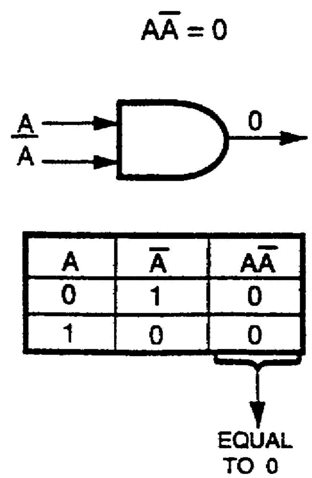
b. Q + R + S

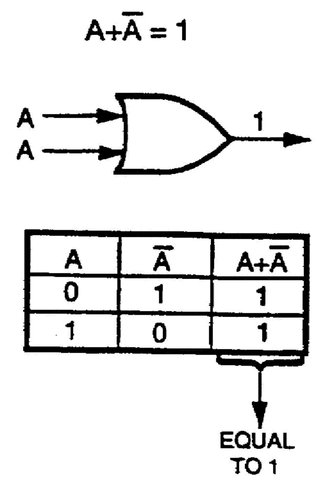
c. (MNP+Q) $L+\overline{K}$

d. WXY +Z

Step by steps solutions on page A-2.

23. When any variable or Boolean expression is ANDed with its complement, the output is 0. When any variable or Boolean expression is ORed with its complement, the output is 1. This is called the law of COMPLEMENTS, as shown by the logic diagrams and corresponding truth tables below.



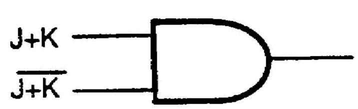


LAW OF COMPLEMENTS

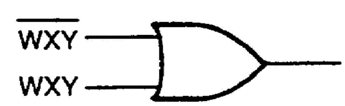
Examples of the LAW OF COMPLEMENTS are: $A\overline{A} = 0$; $JRC + \overline{JRC} = 1$, T+ T = 1; $\overline{(DR + H)}(DR + H) = 0$; etc.

Write the outputs for the logic diagrams below.

a.



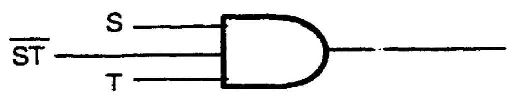
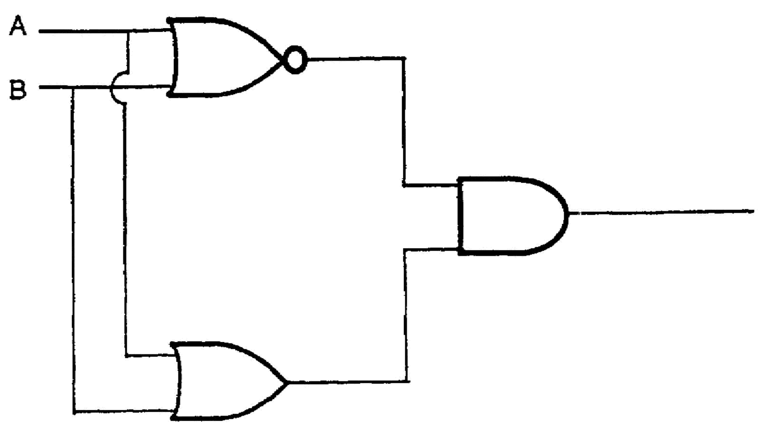
b.



a. 0

b. 1

24. Whenever any variable or Boolean expression is ANDed with its complement, the output is ________.

0	25. Whenever any variable or Boolean expression is ORed with its complement, the output is _________.
1	26. Using the law of COMPLEMENTS, state the output for each of the following expressions or logic diagrams: a. FLR + $\overline{FLR}$ b. $JK\overline{JK}$ c. $\overline{G}$+G d.  e. 

a. 1

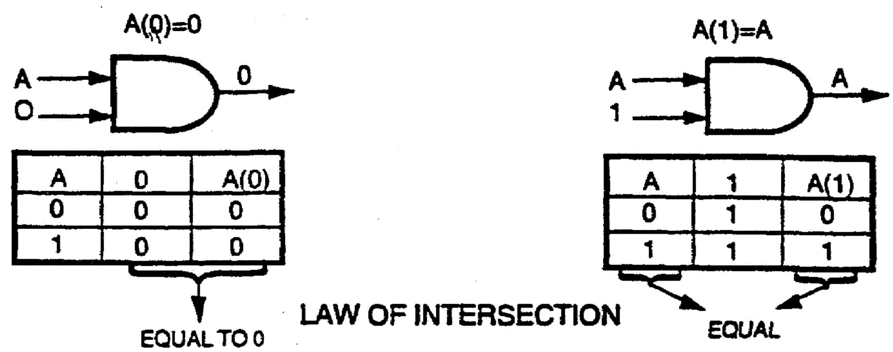
b. 0

c. 1

d. 0

e. 0

27. Another aid in simplifying Boolean expressions is the law of INTERSECTION: A(0) = 0 and A (1) = A, illustrated below.



Complete the following truth tables for the law of INTERSECTION.

a.

A	1	A(1)
	1	
	1	

b.

A	0	A(0)
	0	
	0	

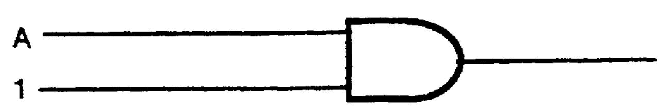
a.

A	1	A(1)
0	1	0
1	1	1

b.

A	0	A(0)
0	0	0
1	0	0

28.

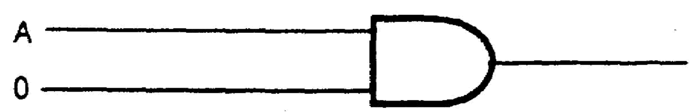


In the logic diagram above, when input A is at the 1 level, the output is _______; when input A is at the 0 level, the output is _______.

1

0

29.



In the logic diagram above, when input A is at the 1 level, the output is _______; when input A is at the 0 level, the output is _______.

0
0

30. The substitution method may be applied as an aid in understanding the basic laws of Boolean Algebra. For example, the law of INTERSECTION, A(0) =0, could readily be applied to simplify the following Boolean expression:

TPZ (0) = 0

Substitute A for the term TPZ, and the expression becomes A(0) =. From page 17 it is found that A(0) =0, according to the law of INTERSECTION; therefore, TPZ(0) = 0.

Using the law of INTERSECTION below, simplify the following Boolean expressions:

A(1) =A   A(0) =0

a. B(1) = ?

b. 0 (D+$\overline{F}$+RS) = ?

c. (BC+C) 0 = ?

d. (Y+Z) 1 = ?

a. B

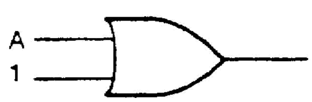
b. 0

c. 0

d. (Y+Z)

31. The law of UNION is similar to the law of intersection, with the exception that the law of UNION applies only to OR logic, as shown below.

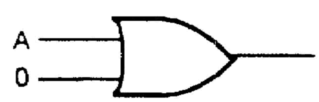
A+1 = 1



A	1	A+1
0	1	1
1	1	1

EQUAL

A+0 = A



A	0	A+0
0	0	0
1	0	1

EQUAL

LAW OF UNION

31. (Continued)

Complete the following truth tables for the law of UNION.

A	1	A+1
	1	
	1	

a.

A	0	A+0
	0	
	0	

b.

---

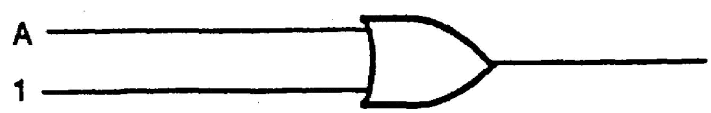
a.

A	1	A+1
0	1	1
1	1	1

b.

A	0	A+0
0	0	0
1	0	1

32.

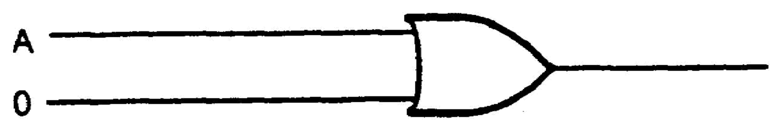


In the logic diagram above, when input A is at the 1 level, the output is ______; when input A is at the 0 level, the output is _______.

1

1

33.



In the logic diagram above, when input A is at the 1 level, the output is _______; when input A is at the 0 level, the output is _______.

1

0

34. A+1=1 A + 0 =A

Using the law of UNION above, simplify the following Boolean expressions:

a. 0+C=? b. 1 +ABE=?

c. QC+0=? d. LM+NP+1 =?

NOTE: The substitution method used in frame 30 may be applied as an aid in simplifying b, c, and d above. Use substitution

	whenever necessary in the remainder of this program.

a. C

b. 1

c. QC

d. 1

35. The laws which have been covered, up to this point, are

IDENTITY:	A=A	$\bar{A} = \bar{A}$
COMMUTATIVE:	AB = BA	A+B=B+A
ASSOCIATIVE:	A(BC) = ABC	A+(B + C) = A + B + C
IDEMPOTENT:	AA = A	A + A = A
DOUBLE NEGATIVE	A=A	
COMPLEMENTARY:	AA = 0	A + A = 1
INTERSECTION:	A • 1 =A	A • 0=0
UNION:	A+1=1	A+0=A

36. Simplify the following Boolean expressions, using the basic laws of Boolean Algebra.

a. T+(V-0) +0

b. E + 0 (AF)

c. 0(K+LM+0)

d. DF+(G+G)

a. T

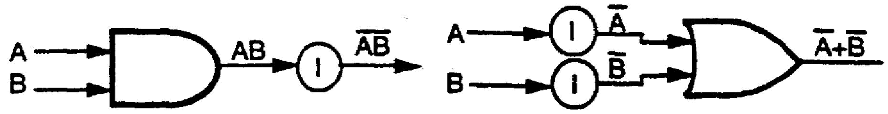
b. E

c. 0

d. 1

37. DE MORGAN'S THEOREM is used to split a vinculum which extends over more than one variable or to join separated vincula into one extended vinculum. DE MORGAN'S THEOREM is illustrated below with the logic diagrams and corresponding truth tables.

$\overline{AB} = \bar{A}+\bar{B}$ (DE MORGAN'S THEOREM)



A	B	AB	$\overline{AB}$	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$\overline{(A+B)} = \bar{A}\,\bar{B}$ (DE MORGAN'S THEOREM)

A	B	A+B	$\overline{(A+B)}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\,\bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

→ EQUAL ←

DE MORGAN'S THEOREM

Whenever vincula are split or joined, change the sign (AND to OR or OR to AND) as indicated below.

$\bar{A}+\bar{B} = \overline{AB}$

When vincula are joined, change the sign.

$\overline{A+B} = \bar{A}\,\bar{B}$

When a vinculum in split, change the sign.

Step by step solutions on page A-3.

37. (Continued)

Split or join the vincula in the Boolean expressions below, using DE MORGAN'S THEOREM.

a. $\overline{J+K+L}$ b. $\overline{R}+\overline{S}+\overline{T}+\overline{Y}$

c. $\overline{DEFG}$ d. $\overline{W}\,\overline{R}\,\overline{Y}$

---

a. $\overline{J}\,\overline{K}\,\overline{L}$

b. $\overline{RSTY}$

c. $\overline{D}+\overline{E}+\overline{F}+\overline{G}$

d. $\overline{W+R+Y}$

38. When the signs are changed in a Boolean expression, group the same variables that were originally grouped. For example:

$$\overline{AB + C} = (\overline{A} + \overline{B})\,\overline{C}$$

Same grouping

Split or join the vincula in the Boolean expressions below, using DE MORGAN'S THEOREM.

a. $\overline{H+SL}$ b. $(\overline{T}+\overline{V})\,\overline{W}$

c. $\overline{R + LM}$ d. $(\overline{S}+\overline{T})(\overline{R}+\overline{P})$

---

a. $\overline{H}\,(\overline{S}+\overline{L})$

b. $\overline{TV+W}$

c. $\overline{R}\,(\overline{L}+\overline{M})$

d. $\overline{ST + RP}$

39. If any variable in a Boolean expression has more than one vinculum over it, the expression is <u>not</u> in the simplest form. For example, $\overline{B} + \overline{CD}$ is not in the simplest form, because variable B has two vincula over it. To simplify Boolean expression B + CD, the <u>longest vinculum is split first</u> by using DE MORGAN'S THEOREM, as follows:

$$\overline{B} + CD = \overline{\overline{B}}\,\overline{CD} = \overline{\overline{B}}\,(\overline{C}+\overline{D})$$

The expression is simplified further by using the DOUBLE NEGATIVE law, as follows:

39. (Continued)

$$\overline{\overline{B}}\,(\overline{C}+\overline{D}) = B\,(\overline{C}+\overline{D})$$

Simplify the Boolean expressions below, using DE MORGAN'S THEOREM, the DOUBLE NEGATIVE law, and the ASSOCIATIVE law.

a. $\overline{(J + K)\overline{(L + M)}}$

b. $\overline{\overline{(R + S)}(D + E)}$

c. $\overline{\overline{DE}+(R + S)}$

d. $\overline{X+Y+\overline{FG}}$

a. $\overline{J}\,\overline{K}+L+M$

b. $R+S+\overline{D}\,\overline{E}$

c. $DE\overline{R}\,\overline{S}$

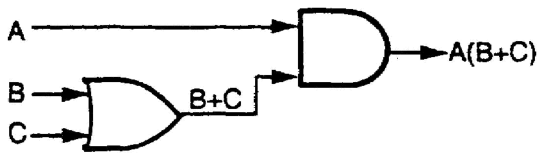
d. $\overline{X}\,\overline{Y}\,FG$

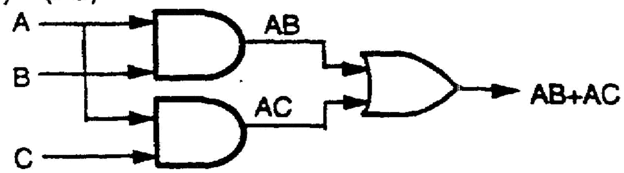
Step by step solutions on page A-3.

40. Often, Boolean expressions <u>cannot</u> be simplified until the expressions have been converted to another form. The DISTRIBUTIVE law is used to convert Boolean expressions from one form to another. The DISTRIBUTIVE law is illustrated on the following page with the logic diagrams and corresponding truth tables.

40. (Continued)

A(B+C) = (AB) + (AC)



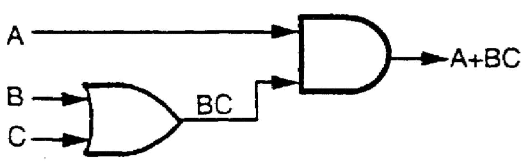


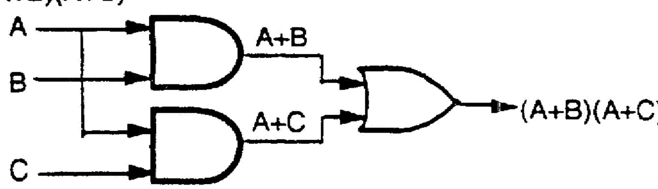
A	B	C	B+C	A(B+C)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

A	B	C	AB	AC	AB+AC
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

EQUAL

A+BC = (A+B)(A+C)





A	B	C	BC	A + BC
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

A	B	C	A+B	A+C	(A+B)(A+C)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

EQUAL

DISTRIBUTIVE LAW

To obtain a 1 output from the Boolean expression A(B+C), input A must be at the 1 level, as well as either input B or input C. In the Boolean expression A(B+C), there are two combinations of variables which will produce a 1 output: AB or AC. Therefore, A(B+C) = AB+AC. When an expression takes the form of A(B+C) or AB+AC, it can be converted to the opposite form by using the DISTRIBUTIVE law.

A(B+C) = AB+AC AB+AC = A(B+C)

	40. (Continued) Convert the following Boolean expressions, using the DISTRIBUTIVE law. a. D(E+F+G) b. QR+QS+QT c. V(W+Y+Z) d. JK+JKL+JKM

a. DE+DF+DG b. Q (R+S+T) c. VW+VY+VZ d. JK (1 +L+M)	41. Another form of the DISTRIBUTIVE law is as shown by the lower set of logic diagrams and truth tables in (frame 40). When input A is at the 0 level, the truth table indicates that both input B and input C must be at the 1 level to obtain a 1 output from the Boolean expression (A+B)(A+C). Therefore, (A+B)(A+C) may be expressed as A + BC. This can be proved by applying the basic laws of Boolean Algebra as follows: (A + B)(A + C) BA + AB + AC=BC — CARRYING OUT MULTIPLICATION A + AB + AC + BC — IDEMPOTENT A(1 + B + C) + BC — DISTRIBUTIVE A = 1 + BC — UNION A + BC — INTERSECTION *When the DISTRIBUTIVE law is used, removing a variable by itself will leave a 1, as shown below. A + AB = (A + AB) = A(1 + B) Convert the following Boolean expressions, using the DISTRIBUTIVE law A + BC = (A + B)(A + C) a. K + LM b. (R + S)(R + T) c. TV + X d. J + KLM

a. (K + L)(K + M)

b. R + ST

c. (T + X)(V + X)

d. (J + K)(J + L)
(J + M)

42. Convert the Boolean expressions below, using the DISTRIBUTIVE law.

a. W + ZXY

b. RS + STV + PSX

c. DE(F + G + H)

d. X + RHS

---

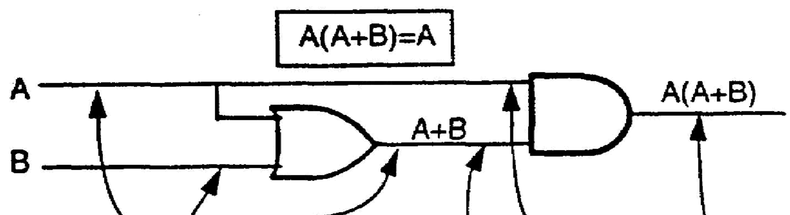
a. (W + Z)(W + X)
(W + Y)

b. S(R + TV + PX)

c. DEF + DEG + DEH

d. (X + R)(X + H)
(X + S)

43. There are two equations pertaining to the law of ABSORPTION: A(A + B) =A and A+(AB) =A. How a single variable effectively absorbs other variables may appear confusing; however, the explanation below, which uses logic diagrams and truth tables, will prove that the equations are, in fact, valid. Since the law of ABSORPTION is often used in simplifying Boolean expressions, carefully study the following:



OR-gate input		OR-gate outputs	AND-gate inputs		AND-gate output
A	B	A+B	A	A+B	A(A+B)
0	0	0	0	0	0
0	1	1	0	1	0
1	0	1	1	1	1
1	1	1	1	1	1

EQUAL IN EVERY CASE

The A input column of the truth table above and the A(A + B) output column are equal (identical) in every case; therefore, A(A + B) is equal to A. In other words, the A variable has effectively absorbed

43. (Continued)

the (A + B) portion of the equation, with the result that A(A + B) = A. Any Boolean expression in the form of A(A + B) can be simplified by using the law of ABSORPTION. Any variable (or quantity) ANDed with an ORed output which contains that variable (or quantity) will absorb the ORed output. (Refer to the logic diagram on the preceding page.) For example, the Boolean expression (T + H + I + S) S can be simplified to a single S variable, because the ORed output (T + H + I + S) contains the same S variable and is effectively absorbed. Boolean expression $AC(AC + \overline{Z}ECA + \overline{X}\,\overline{Y}ACP)$ is simplified to AC, because all the terms within the ORed portion of the expression contain the AC variables and are effectively absorbed, leaving the simplified expression AC. Simplify the following Boolean expressions, using the law of ABSORPTION.

a. $A(A + \overline{W}) =$

b. $AM(MA + \overline{T}HAM) =$

c. (SW + WAST) WS =

d. $(HR + \overline{Z}XRH)\ HR =$

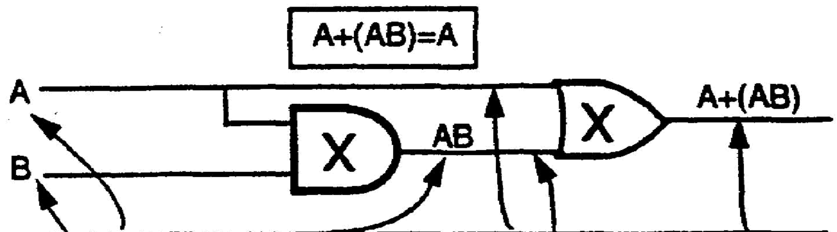
a. A

b. AM

c. WS

d. HR

44. The equation A+(AB) =A, pertaining to the law of ABSORPTION, is shown below with the corresponding logic diagram and truth table.



AND-gate inputs		AND-gate outputs	OR-gate inputs		OR-gate output
A	B	AB	A	AB	A+(AB)
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	1	0	1
1	1	1	1	1	1

EQUAL IN EVERY CASE

As shown by the truth table above, the AND-gate input A and the OR-gate output A+(AB) are equal (identical) in every case. Therefore, A+(AB) is equal to A. Any Boolean expression in the form of A+ (AB) can be simplified by using the law of ABSORPTION. Any variable or quantity ORed with an ANDed output which contains that variable or quantity will absorb the ANDed output. (Refer to the logic diagram above.) For example, the Boolean expression C+(XDC) can be simplified to a single C variable, because the ANDed output (XDC) contains the same C variable and is effectively absorbed.

44. (Continued)

Boolean expression $KM+(A\overline{B}KM+\overline{C}DMK+\overline{F}\,\overline{E}KM)$ is simplified to KM, because all the terms within the ANDed portions of the expression contain the KM variables and are effectively absorbed, resulting in the simplified expression KM.

Simplify the following Boolean expressions, using the law of ABSORPTION.

a. $(\overline{C}D) + D =$

b. $(AQC\overline{M}+FRQA) + AQ =$

c. $X+(XT) =$

d. $EZ+(\overline{A}EZ\overline{T}+EAZT) =$

a. D

b. AQ

c. X

d. EZ

45. Both equations of the law of ABSORPTION, A(A+B) =A and A+(AB) =A can be proved by applying the laws of Boolean Algebra shown below.

A(A+B) =A	
AA+AB=A	DISTRIBUTIVE
A+AB=A	IDEMPOTENT
A(1 +B) =A	DISTRIBUTIVE
A • 1=A	UNION
A=A	INTERSECTION
A+(AB) =A	
A+AB=A	ASSOCIATIVE
A(1 +B) =A	DISTRIBUTIVE
A • 1=A	UNION
A=A	INTERSECTION

Since both equations are equal to A, it is mathematically correct that A(A+B) =A+AB.

Simplify the following Boolean expressions, using the law of ABSORTPION.

a. D+DE

b. K+KL+KM

c. TGNE+T+TI

d. V+W+WX

a. D b. K c. T d. V+W	46. Signs of grouping must be observed when applying the law of ABSORPTION. For example, in the Boolean expression (ABC+AB+D)(A+B), there are two separate groups i.e., (ABC+AB+D) and (A+B). The variables in group (A+B) cannot be used to simplify the (ABC+AB+D) portion of the expression. However, the law of ABSORPTION can be used to absorb effectively a portion of group (ABC+AB+D) as follows: (ABC+AB+D)(A+B) ABSORPTION (AB+D)(A+B) Simplify the following Boolean expression, using the basic laws listed. (MJK+G+K+GGK)(G+KH+LG+K)
a. (MJK+G+K+GK)(G+KH+LG+K)	a. IDEMPOTENT: ______________________________________
b. (G+K)(G+K)	b. ABSORPTION: ______________________________________
c. (G + K)	c. IDEMPOTENT: ______________________________________
d. G + K	d. ASSOCIATIVE: ______________________________________

LESSON

PRACTICE EXERCISE

1. State the use of Boolean Algebra.

2. Multiply the Boolean expressions below.

   a. (B+D)(B+C)

   b. (L+M (P+M)

   c. (AB+C)(D+C)

3. Select examples of the law of IDENTITY.

   a. $CDE = \overline{C}DE$

   b. C = C

   c. (CD) F = (CD) F

   d. $\overline{XYZ} = \overline{XYZ}$

4. Using the COMMUTATIVE law, select the terms below which are equal.

   a. ABC and CBA

   b. F(TP) and F(TZ)

   c. EFGH and GEFL

   d. L(PQ+CD+J+Y) and L(CD+J+PQ+Y)

   e. G(B+A) and (A+B) G

5. Simplify the following Boolean expressions, using the ASSOCIATIVE law.

a. A+(R+S) + (T+V)

b. (DA) N + J(UD) Y

6. Simplify the following Boolean expressions, using the IDEMPOTENT law.

a. $SS+\overline{TX} + \overline{TX}+S+Y$

b. BC+BC+TT

7. Simplify the following Boolean expression, using the laws as listed.

$$X\overline{A} + \overline{A}X + (LTV)\ V$$

a. DOUBLE NEGATIVE:

b. COMMUTATIVE:

c. ASSOCIATIVE:

d. IDEMPOTENT:

8. Simplify the following Boolean expressions, using the law of COMPLEMENTS.

a. $H\overline{H}$ =

b. $F+ \overline{F}$=

c. $ABC + \overline{ABC}$ =

d. $\overline{TCH}TCH$=

9. Simplify the following Boolean expressions, using the law of INTERSECTION.

a. 1 (A) =

b. 1 (A+B+C) =

c. (A) 0 =

d. 0 (A+B+CD) =

10. Simplify the following Boolean expressions, using the law of UNION.

a. A+1=

b. 0 + (AB+CD) =

c. A+0=

d. 1 + (A+B) =

11. Simplify the following Boolean expressions, using DE MORGAN'S THEROEM.

a. $\overline{AB}$

b. $\overline{(A+B)}$

c. $\overline{B} + \overline{(A+G)}$

d. $\overline{(W+G+H)(J+E+S)}$

12. Simplify the following Boolean expressions, using the DISTRIBUTIVE law.

a. AC+ CD

b. (B+A)(N+A)

c. TR+Y

d. DU+PD+DE

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

1. Boolean Algebra is used to manipulate and simplify Boolean expressions.

2. a. BB+BC+DB+DC
   b. LP+LM+MP+MM
   c. ABD+ABC+CD+CC

3. B,C,D

4. A,D,E

5. a. A+R+S+T+V
   b. DAN+JUDY

6. a. $S+\overline{TX}+Y$
   b. BC+T

7. a. $XA + AX + (LTV)\,V$
   b. $\overline{A}X + \overline{A}X + (LTV)\,V$
   c. $\overline{A}X + \overline{A}X + LTVV$
   d. $\overline{A}X + LTV$

8. a. 0
   b. 1
   c. 1
   d. 0

9. a. A
   b. A+B+C
   c. 0
   d. 0

10. a. 1
    b. AB+CD
    c. A
    d. 1

11. a. $A+B$

b. $(\overline{A}\overline{B})$

c. $\overline{B}+(\overline{A}\overline{G})$

d. $(\overline{WGH}) + (\overline{JES}) = (\overline{W}+\overline{G}+\overline{H}) + (\overline{J}+\overline{E}+\overline{S})$

12. a. $C(A+D)$

b. $A+BN$

c. $(T+Y)(R+Y)$

d. $D(U+P+E)$