EDITION A

US ARMY INTELLIGENCE CENTER BASIC CONCEPTS OF BOOLEAN ALGEBRA CONVERSIONS





BASIC CONCEPTS OF BOOLEAN ALGEBRA CONVERSION

Subcourse Number IT 0343

EDITION A

US ARMY INTELLIGENCE CENTER FORT HUACHUCA, AZ 85613-6000

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SUBCOURSE OVERVIEW

This subcourse is designed to teach you to convert logic diagrams to Boolean expressions and to convert Boolean expressions to logic diagrams.

This Subcourse replaces SA 0713.

Prerequisites for this subcourse is IT 0342.

TERMINAL LEARNING OBJECTIVE

ACTION: You will be able to convert logic diagrams to Boolean expressions.

CONDITION: Given the information in this subcourse.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70% on the subcourse examination.

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LESSON 1

CONVERSION OF LOGIC DIAGRAMS TO BOOLEAN EXPRESSIONS

OVERVIEW

LESSON DESCRIPTION:

In this lesson you will learn how to convert logic diagrams to Boolean expressions.

TERMINAL LEARNING OBJECTIVE:

ACTION: Conversion of logic diagrams to Boolean expressions.

CONDITION: Given the information in this lesson.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70% on the subcourse examination.

SPECIAL INSTRUCTIONS:

There are exercises on most pages of this lesson, and some pages have multiple exercises. After you work each exercise, check your solution to the corresponding answer on the following page.

1. A logic diagram which contains only an AND operation or only an OR operation is called a first-order-logic diagram. Output ANDlogic Boolean Inputs symbol expression ABC The diagram above is an example of first-order logic for an AND circuit with three inputs. To convert this diagram correctly to a Boolean expression, first, identify the symbol as an AND-logic symbol. Notice that there are three separate inputs. Be sure to include all the inputs in the output Boolean expression. The correct output Boolean expression for the logic diagram above is written ABC. Write the output Boolean expression for the first-order-logic AND diagram below.





a. M+A+N

4.

b. THIS

- c. I+S
- d. FUN

A logic diagram which contains both an AND and an OR operation

is a second-order-logic diagram.



The diagram above is an example of second-order logic, an AND circuit supplying an OR circuit. The output of each logic symbol must be determined in order to develop an output Boolean expression correctly. Start with the inputs to the first symbol and proceed to the final output. Use the output of each logic symbol as an input to the next succeeding symbol as follows:
Step 1. The first symbol is recognized as an AND gate with two inputs, A and B. The output, AB, in turn, is one of the inputs to the OR gate.
Step 2. The final output from this logic diagram is from the OR gate. An examination of the output expression reveals that it is correctly indicating the output from an OR gate

which had three separate inputs, AB, C, and D.







8. second

9. (H+E) LP

- 10. second
- 11. SWA+B+K



A third-order-logic diagram consisting of two AND gates and one OR gate is illustrated above. The logic diagram can be converted to a Boolean expression by using the same procedure of determining the output of each symbol and working systematically to the final output. Note that proper identity and separation are maintained by using parentheses. If the parentheses had not been used, the output Boolean expression would not have been a correct indication of the logic diagram.

Write the output Boolean expression for the third-order-logic diagram below containing two AND gates and one OR gate.















23a. AB

- b. B+C
- c. AB (B+C)
- 24. (L+M)(L+M)



symbols. Although this diagram is more complex than others previously encountered, it is systematically converted to an output Boolean expression by using the step-by-step procedure illustrated in the diagram. Be careful not to lose track of an inverted input or output. Maintain proper identity and separation by using appropriate marks of separation. A good habit to develop is to doublecheck the results for completeness and accuracy.

Write the output Boolean expression for the logic diagram below.





LESSON 2

CONVERSION OF BOOLEAN EXPRESSIONS TO LOGIC DIAGRAMS

OVERVIEW

LESSON DESCRIPTION:

In this lesson you will learn how to convert Boolean expressions to logic diagrams.

TERMINAL LEARNING OBJECTIVE:

- ACTION: Conversion of Boolean expressions to logic diagrams.
- CONDITION: Given the information in this lesson.
- STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70% on the subcourse examination.

SPECIAL INSTRUCTIONS.

There are exercises on most pages of this lesson, the solution to each exercise immediately follows the exercise. While doing this lesson, use a piece of paper to cover the solution while you work the exercise. After you have completed your work, uncover the solution and check your work.

To convert a Boolean expression to a logic diagram, the technician must first identify the overall type of circuit associated with the expression. This is necessary for two reasons:
 (1) to determine where the expression must first be separated and (2) to determine which logic symbol must first be drawn to represent the final output Boolean expression correctly.

To convert a Boolean expression to a logic diagram, the first step for the technician is to

the _____ ____

of circuit associated with the expression.

identify

overall type

2. To construct a logic diagram from a Boolean expression, begin drawing at the right and work left. If letters in the expression are grouped (by parentheses, brackets, braces, vincula, etc.), <u>first</u>, separate the group from other groups or letters. For example, Boolean expression (A+B)(C+D) indicates that the quantity (A+B) grouped together by parentheses must first be separated from the other group (C+D). Examination of the expression reveals that it is a two-input AND gate with inputs (A+B) and (C+D). If the letter X were substituted for the quantity (A+B) and the letter Y were substituted for the quantity (C+D), the expression would then be represented by XY, a more obvious expression for a two-input AND gate. Substituting single letters for grouped quantities is an aid in determining the overall type of circuit the expression represents.

2.	(Continued)		
	The diagrams below list various typ	es of Boolean expressions, wit	h arrows pointing to the
	first separation points; the overall t	ype of circuit the expression re	epresents; and the first
	logic symbol which must be drawn	(the first logic symbol to the rig	ght) to convert from a
	Boolean expression to a logic diag	ram.	
	Boolean Expression	Overall Type of Circuit	First Logic Symbol
	A+BCD	OR	
	1		
	(B+C)(A+D)	AND	B+C
	B+C (D+E)	OR	B C (D+E)
	C (D+E)	AND	
	1		D+E



2-4





2-5

2.	(Continued)		
	Convert the Boolean expressions be	elow to logic diagrams in the following s	sequence:
	a. Indicate the first separation poin	t/s, with an arrow or arrows;	
	b. state the overall type of circuit; a	and	
	c. draw the first logic symbol.		
	Boolean Expression	Overall Type of Circuit	First Logic Symbol
	A (B+CD)		
	N N25		
	M+NO P		
	(A+BC)(A+C)		
	AB+(X+Z)		

SOLUTION Boolean E	NS: Expression	Overall Type of Circuit	First Logic Symbol
A (B+CD)		AND	A D- B+CD
M+NO P		OR	M DP
(A+BC)(A	+C)	NAND	A+BC A+C
ĀB+(X+Z)		NOR	ĀB X+Z
3. To	o convert a Boolean expression to	a logic diagram, the technic	ian must first
	the		of circuit.
identify			
overall typ	De		

4. A technician is not concerned, to any appreciable extent, in developing original logic. In the course of his normal job, however, he finds it necessary to apply existing logical Boolean expressions to particular circuits for troubleshooting. Boolean expressions are converted to logic diagrams in the same manner previously used to convert diagrams to expressions. The

2-7



5.	(Continued)
	logic symbol for an OR gate with four inputs and label the inputs A, B, C, and D. The
	completed conversion is shown below.
	A A+B+C+D
	Convert the Boolean expression below to a logic diagram.
	B+O+A+T
	B O A T
SOLUT	TION:
6.	a. The Boolean expression ABCD represents a input
	gate.
	b. Convert the Boolean expression ABCD to a logic diagram.
	A ABCD
SOLUT	TION:
	a. four b.
	AND
7.	a. The Boolean expression C+A+J+K represents a input
	b. Convert the Boolean expression G+A+J+K to a logic diagram.



2-9

8. After identifying the overall type of circuit, determining where the expression must first be separated, and drawing the logic symbol which represents the overall circuit, the next step is to convert all groups or letters within the expression systematically until only single-letter inputs remain. For example, the Boolean expression AB (C+D) is converted to a logic diagram as follows:

Step 1. Identify the overall type of circuit.

- a. A B (C+D) this signifies an AND gate.
- b. The AND gate contains three inputs-A, B, and quantity (C+D).
- c. The logic AND symbol with three inputs is shown below.



NOTE: Input C+D is now without parentheses, since it has been separated from the other group and no longer needs a sign of grouping.

Step 2. Identity the input C+D.

- a. C + D- this signifies an OR gate.
- b. The OR gate contains two inputs-C and D.
- c. Draw the logic OR symbol with two inputs as shown below.





11. To convert the Boolean expression D+EF to a logic diagram, it is first necessary to identify the overall circuit as an OR gate (+) with two inputs, D and EF. Input D is separated from input EF by drawing the OR-logic symbol as illustrated below.



The output expression must be separated until only single-letter inputs- remain. When letters are grouped together, as in the example D+EF, the groups must first be separated from other groups or letters. This is accomplished as illustrated in the figure above: input D is separated from input EF. Now, input EF must be identified and put into proper logic-diagram form. EF by itself is identified as an AND gate with two inputs, E and F. Input E is separated from input F by drawing the AND-logic symbol as shown below.



NOTE: Only single-letter inputs remain.

The two symbols are combined to represent the original Boolean expression D+EF by drawing the output of the AND gate as one input (EF) to the OR gate as shown below.



Convert the Boolean expression below to a logic diagram.

EAS+Y

SOLU	
12.	The Boolean expression BAS+C represents ainputinput
two OR	
13.	Convert the Boolean expression below to a logic diagram. BAS+C
SOLUT	
14.	The three-input overall OR-gate Boolean expression, AB+C+D, is converted to a logic diagram by the step-by-step procedure previously discussed. Step 1. Identify the overall type of circuit. a. AB + C + D-this signifies a three-input OR gate. b. The logic OR-gate symbol with three inputs is shown below. AB

0	
14.	(Continued)
	Step 2. Identify input AB.
	a. A B -this signifies a two-input AND gate.
	b. The logic AND-gate symbol with two inputs is shown below.
	A AB
	Step 3. Connect the output of the AND gate (step 2) to the input of the OR gate (step 1.) The completed logic diagram is shown below.
	Convert the Boolean expression below to a logic diagram. GUM+P+Y
	SOLUTION:
15.	The Boolean expression FAS+T+Y represents a
	input gate.
	three
	OR

16. Convert the Boolean expression below to a logic diagram.



SOLUTION:



17. The Boolean expression (AB+C) D is identified as an overall two-input AND gate. Note that the quantity (AB+C) which indicates a two-input OR gate must be treated as a separate quantity when converting from the Boolean expression to a logic diagram. The expression is converted to a logic diagram, following a step-by-step procedure as shown below.

Step 1. Identify the overall type of circuit.

a. (AB+C) D-this signifies a two-input AND gate.

b. The logic AND-gate symbol with two inputs is shown below.



- Step 2. Identity the quantity AB+C.
 - a. AB + C-this signifies a two-input OR gate.
 - b. The logic OR-gate symbol with two inputs is shown below.





18.	The Boolean expression (B+IR) D represents an overall input
	two
	AND
19	Convert the Boolean expression below to a logic diagram
10.	(B+IR) D
SOLU	TION
	BB+IR
20.	When converting a Boolean expression to a logic diagram, pay particular attention to the
	signs of grouping; i.e., parentheses, brackets, braces, vincula, etc. Never separate factors
	within a group until that group has been separated from the rest of the expression. With
	this in mind, the Boolean expression ($R+S$) $T+V$ is systematically converted to a logic
	diagram as follows:
	Step 1. Identify the overall type of circuit.
	a. (R+S) T + V-this signifies a two-input OR gate.
	b. The logic Or-gate symbol with two inputs is shown below.
	(R+S)T (R+S)T+V
	v



	SOLUTION:
	$\begin{array}{c} B \\ O \\ Y \\ \end{array} \end{array} $
	s
21.	The Boolean expression S+T (A+G) represents an overall input
	gate.
	two
	OR
22.	Convert the Boolean expression below to a logic diagram.
	S+T (A+G)
SOLL	JTION:
	S S+T(A+G)
	A A+G / (A+G)
	G
23.	The vinculum is a sign of grouping. When a Boolean expression contains a vinculum,
	such as $\overline{AB}C+DE$, observe the following steps when converting to a logic diagram:
	a. First, separate the groups until the group or letter which has the vinculum over it is
	isolated from the rest of the expression. For example, in the Boolean expression
	ABC+DE, the portion of the expression which contains the vinculum is isolated as
	follows:



	SOLUTION:
24.	The Boolean expression A+BCD represents an overall input
	gate.
	two
	OR
25.	Convert the Boolean expression below to a logic diagram.
	A+BCD
	SOLUTION: A A+BCD
26.	When more than one vinculum covers part of an expression, such as $(\overline{AB}+C)$ D, the
	uppermost vinculum is removed first. Remember that nothing under a vinculum can be
	separated until the vinculum has been removed by using the appropriate logic symbol.
	The Boolean expression ($\overline{\overline{AB+C}}$) D is converted to a logic diagram as follows:
	a. The expression is first identified as a two-input AND gate containing inputs $\overline{(AB+C)}$ and
	D. Draw the logic AND symbol and label the inputs as shown below.
	AB+C (AB+C)D









