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## Fundamentals of Electricity Course

## BASIC ELECTRICITY PART I

*SUBCOURSE MM0703

US Army Ordnance, Missile and Munitions Center and School

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## INTRODUCTION

This is the second of three subcourses on basic electricity. The first subcourse is a review of the mathematics necessary for the study of electricity. This subcourse includes the electron theory, magnetism, inductance, capacitance, and alternating and direct currents. The third subcourse includes resonance, filters, generators, motors, and transformers.

As a missile repairer, an understanding of basic electricity is a must for doing your job. Whether you are new to missile repair or have been in the field for a while, this subcourse will help you sharpen your skills and increase your knowledge.

## Supplementary Requirements

There are no supplementary requirements in material or personnel for this subcourse. You will need only this book and will work without supervision.

## Credit Hours

Fifteen credit hours will be awarded for the successful completion of this subcourse--a score of at least 75 on the end-of-subcourse examination.

## Administrative Instructions

Change Sheets. If a change sheet has been sent to you with this subcourse, be sure you post the changes in the book before starting the subcourse.

Errors on TSC Form 59. Before you begin this subcourse, make sure that the information already typed on your TSC Form 59 (ACCP Examination Response Sheet) is correct. You will find the correct subcourse number and subcourse edition number on the front cover of this book. If any of the information on your TSC Form 59 is incorrect, write to:

The Army Institute for Professional Development (IPD)
US Army Training Support Center
Newport News, VA 23628-0001
A new, correctly filled-out form will be sent to you. Do not correct the form yourself or send it to IPD.

Questions, Changes, Corrections. If you have questions about enrollment or other administrative matters, write to IPD. If a change occurs or a correction needs to be made in your status (name, grade, rank, address, unit of assignment, etc.) notify $I P D$ as soon as possible. These kinds of changes or corrections can be sent along on a separate sheet of paper with your completed TSC Form 59.

Correspondence with IPD. In any correspondence with IPD, always write your name, Social Security Number, and the school code of your enrollment on each page.

## Grading and Certification Instructions

When you have completed the subcourse, review any of the material covered that you are not sure of. Then take the end-of-subcourse examination. When you have completed the examination in the book, you must transfer your answers to TSC Form 59. The instructions on the form itself tell you how to mark your answers on it. Follow the instructions carefully.

Once you have transferred your answers to the TSC Form 59, fold the form as it was folded when sent to you. Do not staple or mutilate this form! Place the form in the self-addressed envelope provided and mail it to IPD. No postage is needed. TSC Form 59 is the only material that you are required to return to IPD. If you return it as soon as you have completed this subcourse, you will get your next subcourse sooner.

Grading. The highest score possible on the end-of-subcourse examination is 100. The grade structure for all ACCP subcourses is given below:

| Superior | $95-100$ |
| :--- | :--- |
| Excellent | $85-94$ |
| Satisfactory | $75-84$ |
| Unsatisfactory | $0-74$ |

Your TSC Form 59 will be machine graded, and you will be notified of the results. Your grade on the examination will be your grade for the subcourse. No credit is given for grades below satisfactory (75).

Certificates. When you have completed the subcourse successfully, IPD will send you a subcourse completion certificate. Keep it with your other personal copies of personnel material. Subcourse completion certificates can be used to support accreditation and other personnel actions.

*     *         * IMPORTANT NOTICE * * *

THE PASSING SCORE FOR ALL ACCP MATERIAL IS NOW 70\%.
PLEASE DISREGARD ALL REFERENCES TO THE 75\% REQUIREMENT.

## Lesson 1

## ELECTRON THEORY AND MAGNETISM

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to describe the principles of electron flow, static electricity, conductors, and insulators and discuss basic electrical concepts and principles of magnetism.

Conditions. You will have this subcourse book and work without supervision.
Standard. You must score at least 70 on the end-of-subcourse examination that covers this lesson and lessons 2, 3, and 4 (answer 27 of the 38 questions correctly).

## ELECTRON THEORY

## Basic Concepts of Matter

The electron theory, which is now accepted and used to explain the behavior of electricity, states that electric current consists of electron flow and can be defined as a moving charge. Electricity can be defined, then, as charged matter. Because an electron, even though smaller than the smallest known microscopic organism, has mass and occupies space, it is matter. To understand what an electron is, you need first to understand the structural nature of matter.

Matter is basically composed of two kinds of electricity (positive and negative). The electron is the basic unit of negative electricity and the proton is the basic unit of positive electricity. There is also a neutral particle called the neutron.

## Molecules

Matter exists in three states; solid, liquid, and gas. You are familiar with water in each of its three states; ice, water, and steam. Regardless of its state, all matter is composed of small particles known as molecules. Solids, liquids, and gases differ in the spacing and forces between the molecules. You know it takes more force to separate ice than it does liquid water. The molecules in gas (steam) will separate themselves if left alone. Molecules in any state are in constant motion.

The molecule is the smallest component into which a compound can be sub-divided by physical means. Boiling water subdivides into separate molecules without changing the chemical composition of the molecules. This is a physical change. To subdivide the molecule into its components requires a chemical change.

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Molecules are composed of atoms. A substance which contains atoms of one kind only is called an element, while those containing more than one kind are called compounds or mixtures. When two or more atoms combine, they form a molecule. If these atoms are not all alike, then the substance formed is a compound. oxygen and hydrogen are both examples of an element. When one oxygen atom and two hydrogen atoms unite, they form a molecule of water. Water is a compound.

## Atoms

As mentioned above, molecules are composed of even smaller particles known as atoms. According to the present concept, an atom is one or more negatively charged particles called electrons, revolving at great speeds in regular, circular, or elliptical orbits around a positive nucleus. The nucleus is one or more positively charged particles called protons and a number of uncharged particles called neutrons. A typical atom, lithium, is shown in figure 1-1.


## Figure 1-1. Atomic Structure as Shown by Lithium.

According to present theory, the nucleus of an atom always has the same number of protons in it as it has electrons outside of it. Uncharged particles, neutrons, are found in the nucleus and add weight to the atom. A proton and a neutron have the same weight, and each is approximately 1,845 times heavier than an electron. The difference between the different atoms is in the number and arrangement of the protons and electrons. Atoms of each of the known elements are of a different weight and size and have distinguishing characteristics.

Figure 1-2 shows the three atoms that make up a molecule of water. It consists of two hydrogen atoms and one oxygen atom. In the hydrogen atom, the nucleus contains one proton (+) whose positive charge is balanced by the negative charge of its one electron (-). In the oxygen atom, the nucleus contains eight protons whose positive charge is balanced by the negative charges of the eight electrons.

The hydrogen atom in figure $1-2$ is the lightest of all atoms, with its single proton and single electron. Some of the heavier atoms (those of the heavier metals such as gold) have over 90 electrons. In the normal atom, the sum of the positive charges of the protons in the nucleus equals exactly the sum of the negative charges of the electrons. Such an atom is neutral.


Figure 1-2. Atoms in a Molecule of Water.

## Ions and Ionization

Under normal conditions, atoms are neutral. However, if for some reason a few electrons are torn away from a neutral atom, the atom becomes "charged" and is called a positive ion. Whereas, if the electrons that are torn away from the neutral atom gather on some other neutral atom, that atom becomes negatively charged and is called a negative ion. In other words, an ion is what is left after an electron has been knocked loose from a neutral atom, or what is created after an electron is added to a previously neutral atom.

The process of an atom gaining electrons or losing electrons is called ionization. Any atom or molecule which carries either a positive or a negative charge is ionized.

Some materials, such as table salt and sulfuric acid, become ionized when mixed with water. The solution as a whole, however, remains neutral. Ionization in gases may result from the collision of two gas molecules, by electron bombardment, or by illumination with a certain kind of light.

The protons within an atom are much heavier than the electrons. Therefore, in an atom of gas, the electrons knocked loose when ionization occurs will move much more easily if some electric force is applied than will the much heavier protons. Ionization of gases is important to you because it happens in electronic equipment, such as radio and television receivers. During the study of electron tubes, many of which are similar to those in home radio receivers, you will see that ionization is sometimes desirable and at other times undesirable.

## Static Electricity

Although this course is mainly about charges in motion, a good understanding of static fields will be helpful to you.

That certain objects attract paper and other light materials when rubbed with various kinds of cloth has been known a long time. The early Greeks were familiar with this method of producing what is now called static electricity. They knew that amber, which they called electron, attracted light objects when rubbed with cloth. The English words electron and electricity are derived from this Greek word for amber. A great deal of our early knowledge about electricity was obtained by experiments on charged bodies, or electricity, at rest.

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When static electricity converts to energy, the effects can sometimes be quite startling. Lightning discharges and the crackling sound in a radio receiver are manifestations of static electricity, releasing its stored up energy.

Charged Bodies and the Force Between Them. Bodies can be charged with static electricity various ways. To understand how, you need to know the following. A charged body merely means that the object has more or less than its normal number of electrons. In the uncharged state, each atom has an equal number of electrons and protons; therefore, in order to charge a body positively, it is necessary to remove some of the electrons. When that happens, there will be an excess of protons or positive charges. The electrons, which were removed are now on some other object, causing it to be negatively charged. (Recall both negatively and positively charged atoms are ions.)

It has been proved experimentally that charged bodies act upon each other with a force of attraction when their charges are unlike and a force of repulsion when their charges are like. Thus, the conclusion is that electrons and protons attract each other, that electrons repel other electrons, and that protons repel other protons. This attraction and repulsion may be stated as the following laws:

- Electric charges of like kind repel each other and charges of unlike kind attract each other.
- The forces of attraction and repulsion are directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

While a unit of electrical charge could be taken as the charge associated with an electron or proton, it would not be practical because it is so small. A more practical unit of charge, called a coulomb, is used. It is about equal to a charge of $6.28 \times 10^{18}$ electrons. The coulomb derives its name from Charles A. Coulomb, a Frenchman who reduced the two laws above to the following formula or law:

The law (Coulomb's Law) can be expressed algebraically as:

$$
\mathrm{F}(\text { dynes })=\frac{\mathrm{Q}_{1} \mathrm{Q}_{2}}{\mathrm{Hd}^{2}},
$$

where $Q_{1}$ and $Q_{2}$ represent the charges in electrostatic units (2.1 $x \quad 10^{9}$ electrons), $d$ the distance in centimeters separating them, and $K$ a constant which depends upon the material separating them. F(dyne) means force in the form of dynes. A dyne is that force which will give an acceleration of 1 centimeter per second, during each second, to a free mass of 1 gram.

Electrostatic or Dielectric Field of Force. The region surrounding and between charged bodies is called the electrostatic field of force. Since this force will act through free space or even through a vacuum, it is different from ordinary forces such as those caused by striking a sharp blow or by exerting steady pressure, like the pressure of water on a dam or the pressure of the air inside an automobile tire. These methods of applying force involve some mechanical connecting link. A field of force differs from these in that it requires no physical or mechanical connecting link, but can be applied through space or through a vacuum.

In order to visualize the various properties of fields of force and their relation to electrical phenomena, you can represent them by imaginary lines that show the direction and intensity of the field. Since it is impossible to imagine enough lines to represent all the paths through space along which the force acts, only a few are drawn, and those only in one plane. The force direction is indicated by an arrowhead and the field strength (or intensity) is indicated by the density or number of lines per unit area. The direction of force is the direction a small positive test object moves or tends to move when acted on by the force.

To test the direction of an electric field, the test object would have to be either a small positive charge or a small negative charge, because the force of a dielectric field will act on either. Scientists use a small positive charge for determining the direction of a dielectric field, and so, this subcourse does, too. (A dielectric field is a field of force that exists between two charged bodies.) In other words, the field about an isolated positive charge (figure 1-3), is away from the charge because a positive test charge would be repelled. The field about an isolated negative charge (figure 1-3), is toward the charge, because a positive test charge would be attracted. The field between a positive and negative charge is from positive to negative for the same reason.


Figure 1-3. The Fields of Force About Single Charges.
Note in figures 1-4 and $1-5$ how lines of force apparently repel each other. In figure 1-4, although the two charges are attracted, the lines of force between the two are not parallel but bulge out at the center as if they were repelling each other. Also note that where they bulge at the center, they are in the same direction; that is, from right to left on the paper.


Figure 1-4. Dielectric Field About Two Unlike Charges.


## Figure 1-5. Dielectric Field About Two Like Charges.

In figure 1-5, the lines of force which are in the region between the charges apparently are repelling each other, as you can judge by the direction of their bends. Although you can say, "like charges repel," the law is stated: Dielectric lines of force in the same direction repel each other. In dealing with certain electric phenomena, this rule is very convenient and useful.

If you briskly rub a rubber rod or comb over a piece of fur or woolen cloth a number of electrons from the fur or cloth adhere to the rubber. If you separate the two immediately, the rubber has an excess of electrons (is negatively charged). If you charge two pith balls oppositely by touching one of them with the rubber and the other with the cloth or fur, they will have an attraction for each other, showing that a force is present. See figure 1-6. You have established a dielectric field. If you allowed the bodies to come together after having been pulled apart, the energy expended in separating them would be regained in the form of force of attraction. This means that energy can be stored in a dielectric field.


UNCHARGED PITH BALLS


POSITIVELY AND NEGATIVELY
CHARGED PITH BALLS CHARGED PITH BALLS ATTRACTING EACH OTHER

positively charged PITH BALLS REPULSING EACH OTHER

negatively charged PITH BALLS REPULSING EACH OTHER

## Figure 1-6. Pith Ball Experimentation With Dielectric Fields.

Even if you move the negatively charged rubber rod some distance away from the cloth or fur, a dielectric field still exists in the space around it. You can see it demonstrated by picking up small bits of paper with the rod or by charging
both of the pith balls from it. The pith balls would then show a force of repulsion between them, indicating the pressure of a dielectric field.

If you use an external force to bring the two charged pith balls closer together, work is done, and the force of repulsion is increased due to the decrease of the natural distance between the two charged bodies. The energy used in decreasing this distance (recovered when you remove the external force) will be used in returning the pith balls to their original position. Here again, it is shown that energy is necessary to establish a force and that the recovered energy has been stored in the field.

If you isolate one negatively charged pith ball and bring the negatively charged rubber rod toward it from any direction, a force or repulsion will be present. If the pith ball is positively charged, it will have an attraction for a negative charge in any direction. The conclusion is that a dielectric field entirely surrounds a charged body.

The Electroscope. It has been shown experimentally that an electric charge can be detected because it attracts light objects such as pith balls, bits of paper, etc.

Any device used for detecting electric charges is called an electroscope. In its simplest form, an electroscope consists of a pith ball hanging on the end of a silk thread. By touching it with a body of a known charge, you have an instrument that can detect charged bodies and that can indicate the type of charge (polarity). To illustrate, if you touch the pith ball with a glass rod, which has been rubbed with silk, you charge the pith ball positively. Any other charged body that is brought near the pith ball will repel it if the body is positive or attract it if the body is negative. The force of repulsion or attraction indicates the strength of the field surrounding the charged bodies.

A better and more sensitive device is the leaf electroscope shown in figure 1-7. It is two thin sheets of metal foil (usually gold or aluminum) called leaves, supported by a wire or stem whose ends pass through a block of sealing wax or insulating material to a metal ball or cap. The leaves are usually sealed in a glass container to prevent air currents and moisture from affecting the instrument. The sensitivity of the instrument depends on several factors, the main two being the thickness and the type of material the leaves are made of.


Figure 1-7. Electroscope.

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If the ball receives either a positive or a negative charge, it causes the leaves to spread apart. The leaves spread because like charges repel. When a charge of positive electricity is placed on the leaves, the spread of the leaves will increase when the ball is approached by a positively charged body. On the other hand, a negatively charged body brought near the ball or cap will decrease the spread.

You can place a charge on the leaves by bringing a charged body near, but without making physical contact with, the ball. This is charging by induction. As soon as you remove the charged body, the electroscope is no longer charged unless you provided some means for it to gain or to lose some electrons while the charge was being induced. You can do this by connecting a wire from the electroscope to some neutral conducting object, such as ground. Then, if a charged body is brought near the electroscope, electrons can leave if the charge is negative or enter if the charge is positive. If the wire is disconnected before the charged body is removed, the electroscope will remain charged oppositely to the charge that induced it. This is charging by conduction because the electroscope comes into direct contact with the charged body.

## Conductors and Insulators

With certain materials, electrons can be quite readily separated from the atom. In fact, there is much evidence to show that in some metals there are free electrons. Experiments show either that free electrons unattached to atoms do exist or that there is a free interchange of orbital electrons between adjacent atoms. The effect is the same: at any instant, the metal seems full of free electrons. Such material, called a conductor, offers little opposition to the movement of electrons between atoms. While, in general, all metals are good conductors, silver, gold, copper, and aluminum are particularly good.

Materials (such as rubber, glass, silk, fur, mica, and air) which have few free electrons are classed as insulators. Such materials offer great opposition to the movement of electrons between atoms. Materials mentioned previously, on which a charge can be placed by rubbing with a dissimilar material, are insulators. If the center of a long rubber rod is rubbed with a piece of fur, an excess of electrons will locate in the rod's center instead of spreading immediately over the whole surface of the rod.

If, on the other hand, an excess of electrons could be placed at a point on a conductor of uniform cross section, they would immediately spread evenly over the entire surface of the conductor because of the free movement of the electrons.

Since all materials, to some extent, both permit and oppose the movement of electrons, there is no such thing as a perfect conductor or a perfect insulator. Even though there is no sharp dividing line between conductors and insulators, only good conductors are used as conductors and only good insulators are used as insulators.

## Distribution of Charges on Objects

The surface density of a charge on an object depends upon the shape of the object (figure 1-8). If you were to touch the charged object with a proof plane (a tiny metal plate with an insulated handle), it would remove part of the charge of the object. If you were then to bring the proof plane in contact with an electroscope, the rise of the electroscope's gold leaves could be taken as a measure


Figure 1-8. Distribution of Charges on Different Shaped Objects.
of the charge on the plane, and hence, the charge density of the object. In this way, you would find that the density of the charge on the outside of a sphere is uniformly distributed. On charged objects other than spheres, the greatest density of charge is found on the part which has the greatest curvature or sharpest point. Thus, if a tear-drop-shaped object were charged, the intensity of the electric field would be greatest in the region of the sharp point. The sharper the point, the lower the breakdown voltage for a given separation and the sooner a spark will jump across the gap. That's why lightening rods and some spark gaps are shaped the way they are.

## Electrostatic Shielding

If you were to take a hollow spherical conductor with a hole in it, you would find that, regardless of the amount of charge on the outer surface, there would be no charge on the inner surface. You could prove this statement by inserting a proof plane into the charged sphere, making contact with the inner surface, removing the proof plane, and testing it with an electroscope. If any charge were present on the inner surface, a part of it would be transferred to the proof plane and the electroscope would show the presence of the charge. This experiment has always shown no measurable charge.

This property of a closed conductor is the basis of electrostatic shielding, that is, enclosing circuit elements in metal cans to isolate them from outside electric fields.

## MAGNETISM

There are important relationships between the laws of magnetism and the laws of electrical currents. Generation and transmission of large amounts of electrical energy on an economical basis is one of the most practical of these relationships. A high-speed digital computer is also an example, as well as the meter that measures the amount of electrical power used in your home. Since magnetism is so important to the field of electricity, you need to be familiar with its principles.

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## History and Definition

The knowledge of magnetism is very old. The early Greeks knew that certain stones (lodestones), found in the district of Magnesia in Asia Minor, had the apparent magic property of attracting small bits of iron. The reason was the ore in these stones, $\mathrm{Fe}_{3} \mathrm{O}_{4}$, called magnetite for the region magnesia. Soon any substance that possessed the property of attracting bits of iron was called a magnet. Attracted substances are known as magnetic substances. The phenomenon associated with magnets and magnetic materials is known as magnetism.

Magnets can also be man-made. You do it by stroking a steel bar with one end of a lodestone or a magnet or by placing a steel bar in a coil of wire through which an electric current is passed.

## Forces Between Poles

If iron filings were sprinkled over a permanent magnet, the greatest concentration of filings would be seen near the end of the magnet (as shown in figure 1-9) with practically none near the center. The regions near the ends of the bar are called the poles of the magnet, and the line joining the two poles is known as the magnet's magnetic axis.


## Figure 1-9. Poles of a Magnet.

If a bar is suspended horizontally by a string or mounted on a pivot, it will line up in roughly a north-south direction. Because one end of the bar magnet will always point north, the pole that tends to seek the magnetic north is called the north pole (N). The other, which tends to seek the magnetic south, is called the south pole (S). Thus, the ends of all bar magnets may be marked as north poles or south poles.

Also, if you mounted bar magnets on pivots (figure 1-10) so that they were free to move, you would find that like poles of two magnets would repel each other, whereas unlike poles would attract each other. By using long, slender, magnetized needles, the north and south poles would be sufficiently separated from each other so that the magnitude of the forces between individual poles could be measured. This was first done by the French physicist Charles A. Coulomb in 1785. By suspending a long needle from a brass wire attached to a graduated


Figure 1-10. Repulsion and Attraction of Poles.
scale, he was able to determine how much mechanical torque had to be applied to hold the needle in its zero position when another long magnetic needle was brought near it. Coulomb determined that the force of repulsion between two north poles was inversely proportional to the square of the distance between them. By experimenting with various magnets, he was able to demonstrate that different magnets, when at the same distance, from the suspended magnet, produced different forces. He was able to assign to each magnet a definite pole strength, relative to a standard magnet. He then expressed the results of his investigation as:

$$
\mathrm{F}=\frac{1}{\mu} \mathrm{X} \frac{\mathrm{~m}_{1} \mathrm{~m}_{2}}{\mathrm{~d}^{2}}=\frac{\mathrm{m}_{1} \mathrm{~m}_{2}}{\mu \mathrm{~d}^{2}}
$$

where $m_{1}$ and $m_{2}$ are the strengths of the poles of the two magnets, $d$ is the distance between poles, and $u$ a constant, which depends mainly on the medium between the poles. The quantity $\mu$ is called the permeability of the medium. When the medium is air, $\mu$ is usually considered to be 1 . Therefore, $\mu$ is left out of the calculations and the formula resolves to:

$$
\mathrm{F}=\frac{\mathrm{m}_{1} \mathrm{~m}_{2}}{\mathrm{~d}^{2}}
$$

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As an illustration, compare the forces acting on two bar magnets 15 cm long with their north poles 10 cm apart as shown in figure $1-11$. Assume the pole strength of each magnet to be 400 units.


Figure 1-11. Forces of Repulsion and Attraction.
Solution:

Force of repulsion between north poles equals

$$
\frac{(400)(400)}{(10)^{2}}=1,600 \text { dynes. }
$$

## Magnetic Fields

Characteristics. From the preceding discussion, you have seen that forces act on bar magnets and magnetic materials brought into the surrounding region of another bar magnet. Remember this as you learn the concept of a magnetic field, which is a region wherein magnetic forces act.

A magnetic field surrounds a bar magnet and permeates it. You can see this by placing a glass plate over a bar magnet and sprinkling iron filings on the glass. By tapping the glass, the iron filings will align themselves with the field and will form chains between the north and south poles of the magnet. The chains, referred to as magnetic lines of force, are lines indicating the direction along which a small magnetic compass tends to align itself. Also, it can be seen that the concentrations of iron filings are greatest where the magnetic field is most intense. They have a definite direction and may be thought of as leaving the north pole and reentering the south pole, and then continuing through the magnet from the south pole to the north pole as shown in figure 1-12.

Another interesting effect appears when other magnets are brought into the magnetic field of the first magnet. The alignment of iron filings along the magnetic lines of force between various magnetic pole combinations is shown in figure 1-13.

Although magnetic lines of force are intangible, they have the following six properties:

1. They are continuous and always form closed loops.
2. They have a tension (along the direction of the lines) which tends to shorten them. Thus, when two unlike poles are brought near each other, the lines of force existing between them are brought closer together.


Figure 1-12. Magnetic Field About a Bar Magnet.


Figure 1-13. Shapes of Magnetic Fields.

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3. They never cross one another.
4. They are conducted by all materials.
5. When from like poles, they tend to push one another apart when the poles are brought near each other.
6. They concentrate in magnetic materials.

Strength of Magnetic Fields. If you place a magnetic pole of strength in a magnetic field, then from the definition of a magnetic field, a force will act upon the pole. The magnitude and direction of this force will vary from point to point, which means that the magnetic field must have a definite direction and magnitude at each point in space. The direction of a magnetic field is that of the force acting upon an isolated north pole.

The intensity of the magnetic field at any point is defined as the force that would be exerted upon a unit north pole if situated at that point. The intensity of a magnetic field in which a unit magnetic pole experiences a force of one dyne is defined as an oersted (after Hans C. Oersted, a Danish physicist). From this definition, if at any point in a magnetic field a pole strength of $M$ units has a force of $F$ dynes acting upon it, the field intensity $H$ at that point in oersteds will be:

$$
\mathrm{H}=\frac{\mathrm{F}}{\mathrm{M}} \text { oersteds. }
$$

Thus, if an isolated pole of 40 units strength, placed at some point in a magnetic field, is acted upon by a force of 240 dynes, the field intensity at that point is:

$$
\mathrm{H}=\frac{\mathrm{F}}{\mathrm{M}}=\frac{240}{40}=6 \text { oersteds. }
$$

Flux density is a measure of the lines of force per unit of area. The unit of flux density, represented by a letter $B$, is the gauss. One line of force is known as a maxwell. One maxwell per square centimeter represents a flux density of one gauss. All of these terms will occur again in your study of electricity.

## Terrestrial Magnetism

A suspended magnet, in orienting itself in a particular direction at every point on or near the earth, shows that the earth is surrounded by a magnetic field. You could think of the distribution of this field as being produced by a huge bar magnet within the earth, located about $17 \varnothing$ away from the earth's axis and having a length much less than the earth's diameter. See figure 1-14.

Since the magnetic pole is away from the geographic north pole, the compass will not point true north (geographic north) over most of the earth's surface. The angle the compass makes with the geographical meridian is called the variation of the compass (declination).

The direction of the earth's magnetic field is not horizontal, except at the magnetic equator, as you could see if you balanced a magnetized needle on a horizontal spindle. This angle between the magnetized needle and the horizontal is called the inclination or angle of dip. The angle of dip increases from zero at the magnetic equator to $90^{\circ}$ at the magnetic poles.


Figure 1-14. Earth's Magnetic Field.

The earth's magnetic field does not remain the same year after year. There are daily, annual, and secular (a period of 960 years) changes. Much work has been done in attempting to explain terrestrial magnetism, but too little is known about the magnetic sources within the earth and atmospheric currents to establish a satisfactory theory of the earth's magnetism.

## Theory of Permanent Magnets

If you took a piece of unmagnetized steel and stroked it with a magnet, it would become a permanent magnet. Careful investigation of the process would reveal that no material had been transferred to the bar of steel. The effect apparently takes place upon something already in the steel bar.

Again, if we took a magnetized steel bar and cut it in two pieces, we would then have two magnets (see figure 1-15), and if the process were continued until molecular dimensions were approached, each resulting particle would be a magnet.


Figure 1-15. Permanent Magnets Showing Continuity of Polarity.

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From this you should be able to assume a working hypothesis that the steel bar, even in its unmagnetized condition, possesses magnetic particles of molecular dimensions distributed throughout the bar randomly. The presence of a magnetic field has the effect of aligning these magnetic particles. This is illustrated in figure 1-16 where you can see the difference between an unmagnetized bar, and a permanent magnet. That poles of a magnet are surface effects reflecting internal conditions is called the molecular theory of magnetism. Recent experimental work in atomic structure assigns the magnetic properties of iron to a large magnetic domain within the atom.


UNMAGNETIZED MOLECULES


MAGNETIZED MOLECULES (PERMANENT MAGNET)

Figure 1-16. Molecular Theory of Magnetism.
Two other terms commonly used in the discussion of magnetic theory are permeability, which is a measure of the relative ease with which magnetic lines of force travel within a material, and reluctance, which is the property of a medium to resist the passing of lines of force. Reluctance corresponds to resistance in an electrodynamic circuit.

## Electromagnetism

The discovery by Hans C. Oersted in 1820, that a compass needle is deflected when placed near an electric current was of fundamental importance in that it immediately suggested a connection between electricity and magnetism. The magnet (needle) returned to its original position as soon as the current was zero, or the needle was deflected in the opposite direction if the current was reversed. Since, by definition, a magnetic field is a region where forces act on magnets, you can imply that a magnetic field surrounds the current-carrying conductor. The magnetic field may be represented by line of force as previously mentioned. As with permanent magnets, you can indicate the nature of the field about any shaped conductor with iron filings.

By exploring the field around a very long conductor with a tiny compass needle, you will find that the lines of force are circles with their centers in the wire (see figure 1-17A). A convenient method of remembering the direction of the field about a wire is to recall the left-hand rule. If you mentally grasp the wire with the left hand, holding the extended thumb in the direction of the electron flow, your fingers circle the wire in the direction of the magnetic field.

The polarity of a coil can also be determined by the left-hand rule. Grasp the coil in the left hand so that the fingers follow the direction in which current is flowing. The thumb will point to the north pole (see figure 1-17B). In the case of a circular loop carrying a current, the flux lines are shown as in figure 1-18.

A. CURRENT-CARRYING CONDUCTOR.

B. POLARITY OF A COIL.

Figure 1-17. Left-Hand Rule for Magnetic Fields.


Figure 1-18. Direction of Flux in a Circular Loop.

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The magnetic field of a solenoid is shown in figure 1-19. A solenoid can be made by winding an insulated wire on a cylinder. On exploring the field, either by a compass needle or with iron filings, you will find that the field is quite uniform at the center but, near the ends, the lines of force diverge. Each line of force, however, is a closed loop. The similarity between the field produced by a solenoid and that of a permanent magnet is striking. In fact, a permanent magnet can be replaced by a suitable solenoid as far as exterior magnetic effects are concerned. The best material for solenoids used to make electromagnets is soft iron.


Figure 1-19. Magnetic Field in a Solenoid.
On the basis of experimental data, the following important generalization can be made: Regardless of its origin, a charge in motion will invariably give rise to a magnetic field.

In the discussion of permanent magnets, you learned that the force between magnets was from an interaction between their fields. Hence, if a current has its own magnetic field, there will be a force acting on a conductor carrying a current when it is placed in a magnetic field. This can be illustrated by the following experiment. A solenoid is freely suspended as shown in figure 1-20. A magnet is hung near one end of the solenoid. The magnet is held rigidly, and a current is sent through the solenoid. The solenoid moves toward the magnet. If the current through the solenoid is reversed, the solenoid moves away from the magnet. A force is acting on a conductor carrying current when placed in a magnetic field.


Figure 1-20. Solenoid in a Magnetic Field.

Figure 1-21A shows a useful way for determining the direction of the force exerted on a current-carrying conductor in a magnetic field. Figure 1-21B indicates a current flowing toward you, out of the paper, with its associated magnetic field, superimposed upon a uniform magnetic field. Figure 1-21C shows the effect of the two magnetic fields upon each other. Since the two fields are traveling in the same direction on the left, the resultant field is strengthened. On the right, the two fields are opposing each other and the resultant field is weak-ended. In this case, the net effect is that the conductor will tend to move toward the weaker part of the field.


Figure 1-21. Force on a Conductor in a Magnetic Field.

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Two parallel current-carrying conductors are shown in figure 1-22. The direction of the magnetic lines of force is given by the left-hand rule. If the currents are in the same direction, the two fields cancel in the area between the two conductors. The conductors tend to move in the direction of the weaker magnetic field, and there is attraction between the wires. If the currents are in opposite directions, the two fields add between the wires. The conductors tend to move in the direction of the weaker magnetic field, and there is repulsion.


SAME DIRECTION


## OPPOSITE DIRECTIONS

Figure 1-22. Fields Surrounding Two Adjacent Conductors.
If a conductor is forced to move through a magnetic field, a current is produced in the conductor. There is an easy way to determine the direction of this induced current. Figure $1-23$ shows the relationship between the direction of $a$ magnetic field, the direction of motion of a conductor, and the direction on the induced current. It is called the left-hand rule for induced current.

From the foregoing, the following rules can be formulated:

- Conductors carrying current in the same direction tend to be drawn together; conductors carrying current in opposite directions tend to be repelled from one another.
- All electric circuits tend to take a position that will make their currents parallel and flow in the same direction.



## REVIEW EXERCISES

Circle the letter of the correct answer to each question.

1. What charge will the atom assume if electrons are torn away from a neutral atom?
a. Negative.
b. Positive.
c. Zero.
d. Neutral.
2. What is a substance, the basic component of which is a molecule of unlike atoms?
a. An element.
b. A mixture.
c. Oxygen.
d. A compound.
3. In which direction will the current-carrying conductor in figure 1-24 move?
a. Left.
b. Right.
c. Up.
d. Down.
4. What is an electric current?
a. Moving electrolyte.
b. Moving electrodes.
c. Moving electrolysis.
d. Moving electrons.
5. What action will cause current flow through the conductor in to the page in figure 1-25?
a. Move field to right.
b. Move conductor to left.
c. Move field up.
d. Move conductor up.
6. If conductor $X$ is fixed, in figure 1-26, $W$ will move in what direction?
a. Right.
b. Left.
c. Up.
d. Down.


Figure 1-24.


Figure 1-25.


Figure 1-26.
7. How is the magnetic field around a current-carrying conductor described?
a. Permanent and natural.
b. Temporary-artificial.
c. Permanent-artificial.
d. Temporary and natural.
8. What represents the angle that the compass makes with the geographical meridian?
a. Inclination.
b. Accident.
c. Declination.
d. Incident.
9. What position do electric circuits tend to take so that their currents will satisfy what conditions?
a. Be perpendicular and flow in opposite directions.
b. Be parallel and flow in the same direction.
c. Be parallel and flow in opposite directions.
d. Be perpendicular and flow in the same direction.
10. A bar magnet with a cross section area of 2 sq cm contains 1,000 lines of magnetic force. What is the flux density of the magnet?

```
a. }500\mathrm{ maxwell.
b. }500\mathrm{ gauss.
c. 2,000 maxwell.
d. 2,000 gauss.
```

11. What is the direction of the resultant magnetic field at point $P$ in figure 1-27?
a. $\uparrow$
b. $\quad \rightarrow$
c. $\quad \downarrow$
d. $\leftarrow$


Figure 1-27.
12. A magnetic pole of 50 unit poles strength exerts a force of 200 dynes upon a second pole 5 cm distance in the air from the first pole. What is the strength, in UP (unit poles), of the second pole?
a. 200 .
b. 198 .
c. $\quad 100$.
d. $\quad 6$.
13. Assume that the magnetic poles are concentrated at the ends of the magnets. What force exists between the south poles? See figure 1-28.
a. 4,000 dynes attraction.
b. 4,000 dynes repulsion.
c. 100 dynes attraction.
d. 100 dynes repulsion.


Figure 1-28.
14. What is the force of repulsion, in dynes, between the two north poles in figure 1-29? (Do not consider forces exerted by the south poles.) Pole strength of each magnet is 800 units.
a. 25,600.
b. 25,824 .
c. 12,784 .
d. 6,327.


Figure 1-29.
15. What is the smallest component into which a compound can be subdivided by physical means?
a. An electron.
b. A proton.
c. A molecule.
d. An atom.
16. How is the resultant field affected by lines of force in the same direction in a magnetic field?
a. Reinforced.
b. Canceled.
c. Attenuated.
d. Neutralized.
17. What material is normally used as an insulator?

| a. | Zinc. |
| :--- | :--- |
| b. | Silver. |
| c. | Gold. |
| d. | Mica. |

18. What difference exists between balanced atoms of various elements?
a. Number and arrangement of electrons and protons.
b. Number and size of electrons and protons.
c. Number and weight of electrons and protons.
d. Number and polarity of electrons and protons.
19. What is the unit of measure of magnetic field intensity?
a. Dyne.
b. Gauss.
c. Coulomb.
d. Oersted.
20. What is the resulting action when a conductor is moved rapidly through a continuous magnetic field?
a. The magnetic field will be canceled.
b. The magnetic field will be neutralized.
c. Current will flow in the conductor.
d. Current will not flow in the conductor.

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed four or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

## Lesson 2 CIRCUITS

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to explain what an electric circuit is, define the standard electrical units of measure, and explain and use Ohm's and Kirchoff's laws.

Conditions. You will have this subcourse book and work without supervision.
Standard. You must score at least 75 on the end-of-subcourse examination that covers this lesson and lessons 1, 3, and 4 (answer 29 of the 38 questions correctly).

## AN ELECTRIC CIRCUIT

Before discussing direct current and the laws which apply to it, you need to understand the elements of a direct or alternating circuit as well as electromotive force and resistance. An electric circuit is formed when a source of electrical potential is connected to an electrical device by means of a conductor. Figure $2-1$ shows a simple circuit with circuit components.

## Terminology

Electrical Potential. The potential (in this case, a battery) provides the driving force for the circuit. Since the negative terminal of the battery has


Figure 2-1. Simple Electrical Circuit.

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an excess of electrons, as compared to the positive terminal, the electrons move from the negative to the positive terminal when there is a completed path.

Conductor. The item which provides this path for the electrons is the conductor. The phenomenon which allows this conduction is at the atomic level. Electrons are not readily separated from an atom because they are maintained within the atom by the attraction force of the positive nucleus. However, electrons are known to exist in a free state apart from the atom. Such electrons are called free electrons. Some substances such as copper, silver, gold, and aluminum contain great quantities of these electrons and are good conductors. Although all conductors offer some opposition to current flow, you can assume this opposition is negligible here.

Electrical Device. In order to use the electrons flowing in the conductor, an electrical device must be inserted. As the electrons pass through the device, they activate the device. In figure $2-1$, the electrical device is a lamp.

Switch. A switch allows you to start and stop the flow of electrons. When you close the switch (start electron flow), a closed circuit is formed, and when you open the switch (stopping current flow), an open circuit is formed.

Fuse. If the electron flow in a circuit is too great, it may damage the electrical device or excessively heat the conductors. To prevent this, a fuse can be inserted. Excessive electron flow produces sufficient heat to melt the fuse and open the circuit, turning it off. A fuse may be considered as an automatic safety switch.

## Kinds of Circuits

Series. Series circuit provides only one path for current to flow. So the current flows through the first resistor, then the second, etc. in series.

Parallel. In a parallel circuit, the current has parallel paths through which it can flow. Each resistor has one end connected to the battery, and the other end is connected to the other end of the battery. The line current or total current leaving the battery must equal the sum of the currents entering the parallel branches.

## ELECTRICAL MEASUREMENTS

In order to learn any more about electrical phenomena, you need to learn the five basic electrical quantities and to specify the units by which they are measured. Other definitions will be added as the need arises.

## Coulomb

The unit of electrical charge (the quantity of excess electrons) is called the coulomb (C). It is the charge that you would get by collecting approximately
$6,280,000,000,000,000,000\left(6.28 \mathrm{X} 10^{18}\right)$ free electrons on a single charged body. Although this large unit is seldom used in calculations, it is important because it is the basis for the definition of other units.

## Ampere

The unit of electron flow is called the ampere (amp). If one C of charge passes a given point on a wire in 1 second, then 1 amp is said to flow. In other words, the ampere is a special name given to a $C$ per second. This electron flow is called current and is represented by the symbol I, which is an abbreviation for "intensity of flow."

## Ohm

The practical unit of resistance to electric current flow is called the ohm. It is that resistance in which 1 volt will maintain a current of 1 amp. The symbol for ohms is $\Omega$, which is the Greek letter omega.

## Volt

The unit of electromotive force (pressure) is the volt. It is the force that will cause 1 amp to flow through a resistance of $1 \Omega$. Voltage is represented by the symbol $V$ or $E$.

## Watt

The unit of electrical power is the watt ( $W$ ). There is one watt of power in a $1-\Omega$ resistor in which a current of 1 amp is flowing. The basic formula is $P=$ IE.

## Practical Values of Electrical Terms

Because it is difficult to visualize the size of these units from the definitions, the following information may be helpful. You can get a better idea of the size of the $C$ by imagining that, if a charge of 6.8 millionths of a C were placed 1 ft from a similar charge, there would be a repelling force of 1 lb. acting between them. A 100 -W light takes about 1 amp of current. The resistance of an electric toaster or flat iron is about $25 \Omega$.

## ELECTROMOTIVE FORCE

As already stated, one requirement of a circuit is a difference of potential or electromotive force (EMF). The EMF may be two unequally charged bodies. If a conductor is placed between them, a current will flow (fig 2-2) for only an instant. The difference in potential is then neutralized. To have an effective current, an EMF must be provided that will remain constant for relatively long periods of time. Batteries or generators are often used for this, with the output terminals being the two unequally charged bodies.


## Figure 2-2. Electromagnetic Force.

## CURRENT

To understand the electron flow produced by an EMF, consider the behavior of electrons in a copper wire. At any instant, the copper wire has free electrons drifting about within the wire. These electrons are attached to the positively charged body and repelled by the negatively charged body. The movement of these free charges through the copper wire makes an electric current. Current flow, as visualized in the electron theory, is illustrated in figure $2-3$. AB is a conductor, composed of atoms (enlarged many, many thousands of times in the illustration), each of which is made up of electrons revolving about its center (including free electrons around the outer orbits). When an EMF is impressed across AB, electrons tend to transfer from atom to atom, from the A end to the B end. This electron movement is current and can be defined as a moving charge.

## RESISTANCE

Resistance is produced by any device in a circuit which offers a continuous opposition to current flow. Energy in some form is dissipated when the EMF overcomes this resistance and causes current to flow. In many cases, resistance is added in a circuit, not to dissipate useful energy, but to improve overall operation. These types of resistors are classified as wire-wound or carbon. The wire-wound variety is made of resistance wire such as nichrome, German silver, or manganin. Carbon resistors are made by mixing powdered carbon with some suitable binder. The mixture is molded into the proper shape. Resistors, can be fixed, adjustable, or variable.

## Fixed

Fixed resistors are used to introduce a constant value of resistance into a circuit. The resistor's size and construction is determined by the amount of power it dissipates without damage to itself. For low-power requirements, small carbon or metalized resistors are used; where heavier dissipation is required, larger resistors of wire-wound construction are used.


Figure 2-3. Electron Behavior.

## Adjustable

Adjustable resistors are used to adjust the value of the resistance in a circuit to compensate for changes in resistor values due to age. The adjustable resistor is usually wire-wound and has one or more sliding collars which may be moved along the resistance element to the desired resistance value. It is then clamped into place and remains fixed until the value changes when it must be readjusted.

## Variable

Variable resistors are used when a resistance requires frequent change.
The volume control on your radio is an example of this. Depending on the power

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requirements, variable resistors are either carbon or wire-wound. The resistance element of the variable resistor is usually circular. The sliding tap (arm), which makes contact with it, has a knob and a shaft with which the resistance can be smoothly varied. Variable resistors are called rheostats or potentiometers.

## LAWS

Kirchoff's Laws used with Ohm's Law makes the solutions of many problems very simple. Application of Ohm's Law is the same for any circuit. Application of Kirchoff's Laws is shown for all three kinds of circuits; series, parallel, and series-parallel.

## Ohm's Law

Ohm's Law states; The current in a circuit is directly proportional to the applied potential (voltage) and inversely proportional to the resistance. It is written mathematically as follows:

$$
I=\frac{E}{R}
$$

where $I=$ current in amps,
$\mathrm{E}=\mathrm{EMF}$ in V ,
$\mathrm{R}=$ resistance in $\Omega$.

This law is based on the findings of George Simon Ohm in 1827. It shows that the amount of current that flows in a circuit increases if the potential applied is increased and decreases if the resistance is increased. It also shows that current decreases when the potential applied decreases, and increases if the resistance decreases. To further understand Ohm's Law, you have to understand the relationships between current, voltage, resistance, and power. Review the paragraph, Electrical Measurements, if you are unsure. Table 2-1 shows Ohm's Law as applied to direct current (DC) and alternating current (AC) circuits. The next lesson is on AC and DC.

The following are some applications of Ohm's Law.

Look at figure 2-4. The EMF is 24 V and the resistance is $60 \Omega$. Solve for current.

Step one--copy formula for I:

$$
I=\frac{E}{R}
$$

Step two--substitute known values:

$$
\mathrm{I}=\frac{24 \mathrm{~V}}{60 \Omega}
$$

Step three--perform indicated division:

$$
I=.4 \mathrm{~A} .
$$

(Note, in equations, amp can be further abbreviated to A.)

Table 2-1. Ohm's Law.

| Known Values | Formulas For Determining Unknown Values of: |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\underset{\text { Current }}{\text { I }}$ | R <br> Resistance | E Voltage | P <br> Power |
| I and R | - | --- - | E $=1 \mathrm{R}$ | $P=I^{2} \mathrm{R}$ |
| I and E | ---- | $\mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}$ | - - - - | $\mathbf{P}=\mathrm{IE}$ |
| $I$ and $P$ | ---- | $\mathbf{R}=\frac{\mathbf{P}}{\mathrm{I}^{2}}$ | $E=\frac{P}{I}$ | ---- |
| R and E | $\mathrm{I}=\frac{\mathrm{E}}{\mathrm{R}}$ | ---- - | - - - - | $\mathrm{P}=\frac{\mathrm{E}^{2}}{\mathrm{R}}$ |
| R and P | $I=\sqrt{\frac{P}{R}}$ | ----- | $\mathrm{E}=\sqrt{\mathrm{PR}}$ | --- |
| $\mathbf{E}$ and $\mathbf{P}$ | $\mathrm{I}=\frac{\mathrm{P}}{\mathrm{E}}$ | $\mathrm{R}=\frac{\mathrm{E}^{2}}{\mathbf{P}}$ | --- | --- |



Figure 2-4. A Series Circuit.
Suppose in figure 2-4, the I is. 4 A and the EMF is 24 V . Solve for resistance. $\mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}=\frac{24 \mathrm{~V}}{.4 \mathrm{~A}}=60 \Omega$

Suppose in figure 2-4, the $R$ is $60 \Omega$, and the $I$ is. 4 A. Solve for EMF.
$\mathrm{E}=\mathrm{IR}=.4 \mathrm{X} 60 \Omega=24 \mathrm{~V}$.

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When current flows through a resistor, a voltage is developed across the resistor. The voltage developed on a resistor is often called a voltage drop. Ohm's Law applies to this voltage drop. Here are some examples.

Suppose $\mathrm{R} 1=20 \Omega$ and $\mathrm{I}_{\mathrm{R} 1}=.4 \mathrm{~A}$. Solve for $\mathrm{E}_{\mathrm{R} 1}$, the voltage drop on R 1 .

$$
\mathrm{E}_{\mathrm{R} 1}=\mathrm{I}_{\mathrm{R} 1} \times \mathrm{R}_{1}=.4 \mathrm{~A} \times 20 \Omega=8 \mathrm{~V} .
$$

Suppose $\mathrm{E}_{\mathrm{R} 2}=60 \mathrm{~V}$, and $\mathrm{R} 2=30 \Omega$. How much is $\mathrm{I}_{2}$ ?

$$
I_{2}=\frac{E_{2}}{R_{2}}=\frac{60 \mathrm{~V}}{30 \Omega}=2 \mathrm{~A}
$$

Suppose $\mathrm{E}_{\mathrm{R} 3}=20 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{R} 3}=2 \mathrm{~A}$. How much is R3?

$$
\mathrm{R}_{3}=\frac{20 \mathrm{~V}}{2 \mathrm{~A}}=10 \Omega
$$

## Kirchoff's Laws

Kirchoff's Laws are also used to solve for voltages, currents and resistances.
Current. Kirchoff's Law for current is that the sum of the currents entering a point is equal to the sum of the currents leaving a point.

In a series circuit there is only one path for current to flow, so $I_{T}=I_{R 1}=$ $I_{R 2}=I_{R 3}$ etc.

In figure $2-4$, if the total current flowing is . 4 A , then $I_{R 1}$, the current flowing in R1, is .4A; $I_{R 2}$, the current flowing in $R 2$, is 4 A ; and $I_{R 3}$, the current flowing in R3, is .4A.

In figure 2-5 is a parallel circuit. In it, according to Kirchoff,

$$
I_{\mathrm{T}}=I_{\mathrm{R} 1}+I_{\mathrm{R} 2} .
$$

Let
$I_{R 1}=.2 \mathrm{~A}$
and
$I_{R 2}=.6 \mathrm{~A}$.
Then
$I_{T}=.2 \mathrm{~A}+.6 \mathrm{~A}=.8 \mathrm{~A}$.
In the series-parallel circuit, shown in figure $2-6$, $R 3$ is a series resistor. R1 is in parallel R2. R3 is called a series resistor because it is the only path through which line current (total current) can flow into the circuit. R1 and R2 make up a parallel combination. The current leaving R3 enters R1 and R2, two parallel paths for current to flow. That is, $I_{3}=I_{T}$ and $I_{1}+I_{2}=I_{T}$.

In figure 2-6,
let
$I_{T}=.5 \mathrm{~A}$,
then
$I_{3}=.5 \mathrm{~A}$.


Figure 2-5. A Parallel Circuit.


Figure 2-6. A Combination Series-Parallel Circuit.
Then let
$I_{1}=.3 \mathrm{~A}$
and

$$
I_{2}=.2 \mathrm{~A} .
$$

Then

$$
I_{T}=I_{1}+I_{2}=.3 \mathrm{~A}+.2 \mathrm{~A}=.5 \mathrm{~A}
$$

Voltage. Kirchoff's Law for voltage is that the sum of the voltages in a loop equals the applied voltage (EMF). A loop is one complete path for current flow from the negative side of the battery, through the circuit, and back to the positive side of the battery.

For the series circuit in figure 2-4: $E_{R 1}+E_{R 2}+E_{R 3}=E_{b}$ ( $E_{A}$ or EMF).
For example,
let
$\mathrm{E}_{\mathrm{R} 1}=8 \mathrm{~V}$,
$\mathrm{E}_{\mathrm{R} 2}=12 \mathrm{~V}$,
$\mathrm{E}_{\mathrm{R} 3}=4 \mathrm{~V}$.
Then
$\mathrm{E}_{\mathrm{A}}=\mathrm{E}_{\mathrm{R} 1}+\mathrm{E}_{\mathrm{R} 2}+\mathrm{E}_{\mathrm{R} 3}=8 \mathrm{~V}+12 \mathrm{~V}+4 \mathrm{~V}=24 \mathrm{~V}$.
For the parallel circuit in figure 2-5: $\mathrm{E}_{\mathrm{R} 1}=\mathrm{E}_{\mathrm{A}}$ because R 1 is the only resistor in its loop. Likewise $E_{R 2}=E_{A}$ for $R 2$ is the only resistor in its loop. So $E_{A}$ $=\mathrm{E} 1=\mathrm{E} 2$.

For example, let $\mathrm{E}_{\mathrm{A}}=24 \mathrm{~V}$.
Then
$\mathrm{E}_{\mathrm{R} 1}=24 \mathrm{~V}$,
and

$$
E_{R 2}=24 V
$$

For the series-parallel circuit of figure 2-6: $E_{R 3}+E_{R 1}=E_{A}$ because $R 3$ and $R 1$ are the only resistors in their loop. Likewise $E_{R 3}+E_{R 2}=E_{A}$.

For example,
let
$\mathrm{E}_{\mathrm{R} 3}=15 \mathrm{~V}$
and
$E_{R 2}=30 \mathrm{~V}$.
Then
$\mathrm{E}_{\mathrm{A}}=\mathrm{E}_{\mathrm{R} 3}+\mathrm{E}_{\mathrm{R} 2}=15 \mathrm{~V}+30 \mathrm{~V}=45 \mathrm{~V}$.
Resistance. Kirchoff's Laws for resistance follow from the laws for current and voltage. Actually Kirchoff stated only two laws, one for voltage and one for current. The principles for resistance are mathematical derivations from Kirchoff's Laws for voltage and current.

In a series circuit, total resistance equals the sum of the individual resistances. That is, in figure $2-4, R_{T}=R 1+R 2+R 3$.

For example in figure 2-4,
let
$R_{1}=20 \Omega$,
$R_{2}=30 \Omega$,
and

$$
\mathrm{R}_{3}=10 \Omega .
$$

Then

$$
\mathrm{RT}=\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{3}=20 \Omega+30 \Omega+10 \Omega=60 \Omega
$$

In a parallel circuit, the total resistance equals the reciprocal of the sum of the reciprocals of the resistors. (Recall that the reciprocal of a number is one divided by that number.)

So, in figure 2-5,

$$
\mathrm{R}_{\mathrm{T}}=\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}}}
$$

This is called the reciprocal formula for total resistance of resistors in parallel.

For example in figure 2-5,
let

$$
\mathrm{R}_{1}=120 \Omega
$$

and

$$
\mathrm{R}_{2}=40 \Omega
$$

Then

$$
\begin{aligned}
& \mathrm{RT}=\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}}}=\frac{1}{\frac{1}{120}=\frac{1}{40} \quad=\frac{1}{\frac{1}{120}=\frac{3}{120}}} \\
& \mathrm{RT}=\frac{1}{\frac{4}{120}}=1 \quad \mathrm{X} \frac{120}{4}=\frac{120}{4}=30
\end{aligned}
$$

If there are only two resistors in parallel, then the reciprocal becomes

$$
\begin{aligned}
& \frac{\mathrm{R}_{1} \times \mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \text { as shown here: } \\
& \mathrm{RT}=\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}}}=\frac{1}{\frac{\mathrm{R}_{2}}{\mathrm{R}_{1} \times \mathrm{R}_{2}}+\frac{\mathrm{R}_{1}}{\mathrm{R}_{1} \times \mathrm{R}_{2}}}=\frac{1}{\frac{\mathrm{R}_{2}+\mathrm{R}_{1}}{\mathrm{R}_{1} \times \mathrm{R}_{2}}} \\
& \mathrm{RT}=1 \times \frac{\mathrm{R}_{1} \times \mathrm{R}_{2}}{\mathrm{R}_{2}+\mathrm{R}_{1}}=\frac{\mathrm{R}_{1} \times \mathrm{R}_{2}}{\mathrm{R}_{2}+\mathrm{R}_{1}}
\end{aligned}
$$

This is known as the product-over-the-sum formula for two resistances in parallel.

If all the resistors in parallel are equal, the reciprocal formula becomes $\mathrm{RT}=$ R1 $R_{n}$, where $R 1$ is value of one resistor, and $R_{n}$ is number of resistors.

Proof: In figure $2-5$ if $R 1=R 2$, then the reciprocal formula

$$
\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}}}=\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{1}}}
$$

Now

$$
\frac{1}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{1}}}=\frac{1}{\frac{2}{\mathrm{R}_{1}}}=1 \times \frac{\mathrm{R}_{1}}{2}=\frac{\mathrm{R}_{1}}{2}
$$

This is known as the equal value formula for resistance of equal resistors in parallel.

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Series-parallel circuit resistance is found by adding resistance of series resistors to the equivalent resistance of combinations. Combinations may be resistors in parallel or in series with each other but not with the battery.

The symbol for the equivalent resistance of the combination of $R 1$ and R2 in parallel with each other may be Req1, 2 or $R_{C 1}, 2$ or $R_{p a r} 1,2$. The tags of schematic numbers may be omitted if there is only one combination in the circuit.

In figure 2-6,
let

$$
\mathrm{R}_{1}=100 \Omega
$$

$$
R_{2}=150 \Omega
$$

and

$$
\mathrm{R}_{3}=30 \Omega .
$$

Then

$$
\mathrm{R}_{\mathrm{T}}=\mathrm{R} 1+\mathrm{Req}_{2,3} \text { (Kirchoff's Law). }
$$

Now,

$$
\operatorname{Req}_{2,3}=\frac{\mathrm{R}_{1} \times \mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}}(\text { product }- \text { over }- \text { the }- \text { sum })
$$

Substituting

$$
\operatorname{Req}_{2,3}=\frac{100 \times 150}{100+150}=\frac{15,000}{250}=60 \Omega
$$

So,
$R_{T}+R_{C}+60 \Omega=90 \Omega$.

## POWER

Power is work done. Power is a product of voltage and current. The formulas for power are

$$
\begin{aligned}
& \mathrm{P}=\mathrm{EI} \\
& \mathrm{P}=\mathrm{I}^{2} \mathrm{R} \\
& \mathrm{P}=\frac{\mathrm{E}^{2}}{\mathrm{R}}
\end{aligned}
$$

The most commonly used are EI and $I^{2} R$. $I^{2} R$ losses is a term commonly used by electric power companies transferring power through miles of wire from the generator on a dam to a city.

The basic unit of measurement of power is the watt. Standard units are kilowatt (kW), megawatt (MW), milliwatt (mW), and microwatt ( $\mu \mathrm{W}$ ) .

Power dissipated is power used (consumed) by resistance. It may be in the form of heat, as in a toaster, oven, or iron. The power ratings of components in electrical circuits are often $2-1 / 2$ times as much as would be dissipated in the circuit.

To find total power dissipated by a circuit, whether it is series, parallel, or series-parallel; simply add the powers dissipated by each resistor.

## SPECIAL-CASE PROBLEM SOLVING

Ohm's law, as previously stated, is
Where: I is the current in amperes,
E is the EMF in volts,
$R$ is the resistance in ohms,
$P$ is the power in watts,
$\mathrm{I}=\frac{\mathrm{E}}{\mathrm{R}}, \mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}}, \mathrm{E}=\mathrm{IR}$, then $\mathrm{P}=\mathrm{EI}$

Problem 1. Find the total resistance of the circuit in figure 2-7.
Solution:

$$
\mathrm{R}_{\mathrm{T}}=\mathrm{R} 1+\mathrm{R} 2+\mathrm{R} 3=20+30+10=60 \Omega
$$

Problem 2. Find the total current flowing in the circuit in figure 2-7.

Given:

$$
\mathrm{E}_{\mathrm{b}}=24 \mathrm{~V} \quad \mathrm{R}_{\mathrm{T}}=60 \Omega
$$

Solution:

$$
\begin{aligned}
& \mathrm{I}=\frac{\mathrm{E}}{\mathrm{R}} \\
& \mathrm{IT}=\frac{24}{60}=.4 \mathrm{amp}
\end{aligned}
$$

Problem 3. Find the voltage drops across each resistor in the circuit in figure 2-7.

Solution:
$\mathrm{E}=\mathrm{IR}$
Solving for $\mathrm{E}_{\mathrm{R} 1}, \mathrm{E}=.4 \mathrm{x} 20=8 \mathrm{~V}$.
Solving for $\mathrm{E}_{\mathrm{R} 2}$, $\mathrm{E}=.4 \mathrm{x} 30=12 \mathrm{~V}$.
Solving for $\mathrm{E}_{\mathrm{R} 3}, \mathrm{E}=.4 \mathrm{X} 10=4 \mathrm{~V}$.
Assuming $\mathrm{E}_{\mathrm{b}}=24 \mathrm{~V}, \mathrm{E}_{\mathrm{b}}=\mathrm{E}_{\mathrm{R} 1}+\mathrm{E}_{\mathrm{R} 2}+\mathrm{E}_{\mathrm{R} 3}=8+12+4=12 \mathrm{~V}$.


Figure 2-7. Series Circuit for Problems 1, 2, and 3.

## MM0703, Lesson 2

Problem 4. Find the value of $R 3$ in the circuit of figure 2-8.
Given:
$\mathrm{E}=24 \mathrm{~V}$
$I=.5 \mathrm{amp}$
With E and I known, you can solve for RT.
Solution:
$R=\frac{E}{I}$
$\mathrm{RT}=\frac{24}{.5}=48 \Omega$
Then,

$$
R 3=R_{T}-R 1-R 2=48-10=8 \Omega .
$$

Problem 5. Prove that $\mathrm{E}_{\mathrm{b}}$ in the circuit in figure 2-8 equals 24 V .

```
Given:
    I =.5 amp
    R=48\Omega
```

Solution:
$\mathrm{E}=\mathrm{IR}$
$\mathrm{E}_{\mathrm{b}}=.5 \mathrm{X} 48=24 \mathrm{~V}$

Problem 6. Find the total resistance in the circuit of figure 2-9.

$$
\begin{aligned}
& \text { Given: } \\
& \begin{array}{l}
\mathrm{E}=24 \mathrm{~V} \\
\mathrm{I}=6 \mathrm{amps} \\
\text { Solution: } \\
\mathrm{R}=\frac{\mathrm{E}}{\mathrm{I}} \\
\mathrm{RT}=\frac{24}{6}=4 \Omega
\end{array}
\end{aligned}
$$

Problem 7. Solve for the resistance of $R 3$ in the circuit of figure 2-9. Several methods could be used in this problem. First determine what information is known. Total resistance (solved in problem 6), total current, and the


Figure 2-8. Series Circuit for Problems 4 and 5.
applied voltage are all known values. Referring to Kirchoff's law (current coming out of a branch is equal to the current entering the branch), at point $A$ there are 6 amps of current and at point $B$ there must be this same 6 amps of current. Notice at point A there are three paths for current flow. The same voltage (potential) is felt across each path.

Given:
where

$$
I_{T}=I_{R 1}+I_{R 2}+I_{R 3},
$$

Solution:

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{R} 1}=\frac{24}{10}=2.4 \mathrm{amp} \\
& \mathrm{I}_{\mathrm{R} 2}=\frac{24}{40}=1.2 \mathrm{amp}
\end{aligned}
$$

Then,

$$
I_{R 3}=I_{T}-I_{R 1}-I_{R 2}=6-2.4-1.2=2.4 \mathrm{amp} .
$$

$$
\mathrm{R} 3=\frac{24}{2.4}=10 \Omega
$$

or
$\mathrm{RT}=\frac{1}{\frac{1}{\mathrm{R} 1}+\frac{1}{\mathrm{R} 2}+\frac{1}{\mathrm{R} 3}}=\frac{1}{\frac{1}{10}+\frac{1}{20}+\frac{1}{10}}=\frac{1}{\frac{2}{20}+\frac{1}{20}+\frac{2}{20}}=\frac{1}{\frac{5}{20}}$
$1 \times \frac{20}{5}=4 \Omega$
Problem 8. Find the total power consumed in the circuit of figure 2-9;

Solution.
$\mathrm{P}=1 \mathrm{E}$
$\mathrm{P}_{\mathrm{T}}=\mathrm{I}_{\mathrm{T}}=6 \mathrm{X} 24=144 \mathrm{~W}$,
or

$$
P_{T}=I^{2} R=36 \times 4=144 \mathrm{~W},
$$

or

$$
\mathrm{PT}=\frac{\mathrm{E}^{2}}{\mathrm{R}}=\frac{576}{4}=144 \mathrm{~W},
$$



Figure 2-9. Parallel Circuit for Problems 6, 7, and 8.

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Problem 9. Find the total resistance of the series-parallel circuit in figure 2-10. The first problem is to find the equivalent resistance of $R 2$ and $R 3$, which are in parallel. Once you find this value, the resistance of the branch is referred to as RA then,

Solution:
$\mathrm{R}_{\mathrm{A}}=\frac{\mathrm{R} 2 \times \mathrm{R} 3}{\mathrm{R} 2+\mathrm{R} 3}=\frac{20 \times 20}{20+20}=\frac{400}{40}=10 \Omega$
With the value of RA known, the circuit can be handled as a series circuit in calculating the total resistance:
$\mathrm{R}_{\mathrm{T}}=\mathrm{R} 1+\mathrm{R}_{\mathrm{A}}+\mathrm{R} 4=10+10+20=40 \Omega$.
Problem 10. Find the current flowing in the circuit of figure 2-10;
Solution:
$I=\frac{E}{R}$

$$
\mathrm{I}_{\mathrm{T}}=\frac{\mathrm{E}_{\mathrm{b}}}{\mathrm{R}_{\mathrm{T}}}=\frac{24}{40}=.6 \mathrm{amp}
$$

Problem 11. Find the voltage drop across each resistor or branch in the circuit of figure 2-10;

Solution:

$$
\begin{aligned}
& \mathrm{E}=\text { IR. } \\
& \mathrm{E}_{\mathrm{R} 1}=.6 \times 10=6 \mathrm{~V} \\
& \mathrm{ERA}=.6 \times 10=6 \mathrm{~V} \\
& \mathrm{ER} 4=.6 \times 20=12 \mathrm{~V}
\end{aligned}
$$



Figure 2-10. Series-Parallel Circuit for Problems 9, 10, 11, and 12.

Problem 12. Find the current flow through R2 and R3 (figure 2-10).

$$
\begin{aligned}
& \text { Given: } \\
& \mathrm{I}_{\mathrm{T}}=.6 \mathrm{amp} \\
& \mathrm{E}_{\mathrm{RA}}=6 \mathrm{~V} \\
& \text { Solution: } \\
& \mathrm{I}=\frac{\mathrm{E}}{\mathrm{R}} \\
& \mathrm{I}_{\mathrm{R} 2}=\frac{6}{20}=.3 \mathrm{amp} \\
& \mathrm{I}_{\mathrm{R} 3}=\frac{6}{20}=.3 \mathrm{amp} \\
& \mathrm{I}_{\mathrm{T}}=\mathrm{I}_{\mathrm{R} 2}+\mathrm{I}_{\mathrm{R} 3}=.3+.3=.6 \mathrm{amp}
\end{aligned}
$$

Circle the letter of the correct answer to each question.

1. What is the basic power formula?
a. $\quad \mathrm{P}=\frac{\mathrm{I}}{\mathrm{E}}$
b. $\quad P=\frac{E}{A}$
c. $\quad P=I E$.
d. $\quad P=E A$.
2. What is the unit of electrical power?
a. The ampere.
b. The gauss.
c. The volt.
d. The watt.
3. What symbol represents current?
a. R.
b. I.
c. P.
d. A.
4. What is the unit of electromotive force?
a. The coulomb.
b. The ampere.
c. The watt.
d. The volt.
5. What is the total resistance of the parallel circuit containing resistors of $10 \Omega$ and $15 \Omega$ ?
a. 6 .
b. 10 .
c. 14 .
d. $\quad 18$.
6. What is the electrical resistance formula?
a. $\quad \mathrm{R}=\frac{\mathrm{W}}{\mathrm{I}}$
b. $\quad R=\frac{I}{E}$
c. $\quad R=\frac{E}{I}$
d. $\quad \mathrm{R}=\frac{\mathrm{I}}{\mathrm{W}}$
7. What is the total resistance of a parallel circuit containing five 40- $\Omega$ resistors?
a. 8 .
b. 16 .
c. 32 .
d. 64 .
8. A circuit consumes 5 W of power from a potential of 50 V . What is the resistance, in $\Omega$, of this circuit?
a. 25 .
b. 250 .
c. 500 .
d. 5,000.
9. What potential, in $V$, will cause . 2 amps of current to flow through 1,000 $\Omega$ of resistance?
a. 30 .
b. 200 .
c. 300 .
d. 2,000.
10. What is the total resistance in $\Omega$ in figure 2-11?
a. 40 .
b. 70 .
c. 100 .
d. 130 .


Figure 2-11.
11. How much current, in amps, will flow in the circuit in figure 2-11?
a. . 05 .
b. .5.
c. 5 .
d. 50 .
12. How much power in $W$, is consumed in the circuit in figure 2-11?
a. 2.5.
b. 5.5.
c. $\quad 25$.
d. 55 .
13. How many $V$ are dropped across resistor $R 1$ in figure 2-11?
a. 30 .
b. 32 .
c. 40 .
d. 42 .
14. How much power, in $W$, is dissipated across resistor $R 1$ in figure 2-11?
a. 5 .
b. 10 .
c. 15 .
d. 20 .
15. How much power, in $W$, is dissipated across resistor $R 3$ in figure 2-11?
a. 5.5 .
b. 7.5 .
c. $\quad 9.5$.
d. 11.5.
16. What is the total resistance in $\Omega$ in figure $2-12$ ?

$$
\begin{array}{ll}
\text { a. } & 2.5 . \\
\mathrm{b} . & 5 . \\
\mathrm{c} . & 7.5 . \\
\mathrm{d} . & 10 .
\end{array}
$$

17. How many amps of current flow through resistor R1 in figure 2-12?
a. . 1 .
b. 1 .
c. 2 .
d. 4 .
18. How much power, in $W$, is dissipated across resistor $R 3$ in figure 2-12?
a. 10 .
b. $\quad 20$.
c. 30 .
d. 40 .
19. What is the total power, in $W$, consumed by the circuit in figure 2-12?
a. 40 .
b. 60 .
c. 80 .
d. 100 .


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20. What is the Ohms Law formula for computing current?
a. $\quad I=\frac{E}{R}$
b. $\quad I=\frac{R}{E}$
c. $\quad I=\frac{E}{V}$
d. $\quad I=\frac{V}{E}$

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise solutions. If you missed five or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

## Lesson 3

## DIRECT AND ALTERNATING CURRENTS

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to explain the characteristics of direct and alternating currents; the generation of a sine wave of voltage or current; vector representation of a sine wave; and peak, average, and effective values of alternating current.

Conditions. You will have this subcourse book and work without supervision.
Standard. You must score at least 70 on the end-of-subcourse examination that covers this lesson and lessons 1, 2, and 4 (answer 27 of the 38 questions correctly).

## DEFINITION

## Direct Current

A direct current (DC) exists when a number of electrons move in one direction only, from a point of low potential to a point of high potential. For example, when the circuit of a flashlight is completed by the switch, electrons continue to move from the negative terminal toward the positive terminals through the filament of the bulb, as long as the switch is on.

## Alternating Current

In an alternating-current (AC) circuit, the electrons move through the circuit in one direction for a short period of time, and then move back in the other direction for a like period of time, changing direction at definite intervals. It is this characteristic that gives it its name. The electrons also move toward a point of higher potential, but this point changes from one side of the circuit to the other. In other words, the polarity of the applied alternating voltage changes causing the direction of current flow to change. The two systems are illustrated in figure 3-1.


Figure 3-1. AC and DC Circuits.

## History

In the beginning of the electrical age, only DC existed. It worked well because energy was consumed relatively close to where it was generated. However, as the demand for electric energy increased, energy had to be transmitted over greater distances. Because power is lost in long-distance lines, DC had to be improved upon.

The solution to the problem came on August 31, 1831, when Michael Faraday wound two separate coils of wire upon an iron ring. The ends of one coil were connected to a battery and switch. The ends of the other coil were joined by a long wire, which passed over a compass needle. When the switch was closed completing the battery circuit, the compass needle moved, then oscillated back and forth, and finally stopped in its original position. When the switch was opened, the needle again moved, in the opposite direction, oscillated, and came to rest. This momentary flow of current was completely unexpected, and Faraday had difficulty explaining it. Later he showed that magnetic and electrical actions were reciprocal because it is necessary for electricity to move before a magnetic effect is produced, and it is necessary for magnetism to move (or change) before an electrical effect is produced. Based on this discovery, AC systems were soon developed that greatly reduced this power loss during transmission. Alternating current systems work so well, they are now used almost exclusively in powerlines.

All forms of communications are based upon variations in the AC transmitting medium that can, for instance, mirror the variation in sound waves your voice produces. In order to communicate by telephone or radio, sound waves must be made to cause electrical variations in the transmitter and receiver. Alternating currents can accommodate these electrical variations. Thus can the intelligence in the sound waves be transmitted electrically.

Direct current is simple, but the generation and characteristics of AC are complex. The rest of this lesson is on AC.

## GENERATION OF CURRENT AND VOLTAGE BY AN AC SYSTEM

## Principles

An AC generator converts mechanical energy into electrical energy. It does this by the principle of electromagnetic induction. A review of this principal follows.

In the section on magnetism, you learned that a current-carrying conductor produces a magnetic field around itself. You also learned that when a currentcarrying conductor is placed in a magnetic field, a force acts on the conductor. This force is perpendicular to the conductor and to the lines of force in the magnetic field. Conversely, if a conductor is moved across the lines of force in a magnetic field, current is caused to flow in the conductor. This current flow also results when the magnetic field is moved with respect to the conductor or when both are moved with respect to each other.

The direction of current flow depends upon the relative motion of the conductor and magnetic field. The amount of current depends upon the strength of the field, speed of relative motion of the conductor and field, and the angle at which the conductor cuts the magnetic lines of force (flux).

Even though current flows through the conductor because the conductor is in motion in a magnetic field, current does not just happen to flow. There must be an EMF applied first. Experiments seem to indicate that an EMF is induced into the conductor from the magnetic lines of force when they are broken by the conductor. This induced EMF forces the free electrons within the conductor to move.

The motion is as indicated by the left-hand rule. Recall from lesson 1 that it is used to determine the direction of current in a conductor. This rule is illustrated in figure $3-2$, showing lines of force in a magnetic field.


Figure 3-2. Left-Hand Rule Illustrating Current Flow in a Wire, Moving Through a Magnetic Field and Altering Fields of Force.

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## Generation of AC Voltage

The amount of voltage induced in a coil is dependent upon these factors: strength of the magnetic field, number of turns in the coil, and angle at which the coil cuts the lines of force.

Any movement of the coil in figure $3-3$ would induce a voltage in the coil, since it would be cutting the lines of force between the magnetic poles. In figure 33A, this single turn coil is at rest in position AB. As the coil rotates in a counterclockwise direction from A toward A' in figure 3-3B, the winding of the coil breaks the lines of force at right angles. This induces a maximum voltage into the coil.


## Figure 3-3. AC Generator.

The polarity of the induced voltage depends upon the direction in which the field of force is cut. Side A of the coil cuts the lines of force in an upward direction, and side $B$, at the same instant, cuts the lines of force in $a$ downward direction, producing different polarities. Side A has the negative polarity; side B has a positive polarity. With an external circuit connected to the coil, current would flow from side A through the external circuit to side B. The current inside the coil is from B to A. As the coil rotates from A to A', the angle at which the coil cuts the lines of force is decreased from 90 degrees to 0 degrees. As the angle decreases, the magnitude of the induced voltage decreases until at A'B' the loop is not cutting any lines of force because it is moving parallel to them. At A'B' there is no induced voltage. As the coil continues to rotate, side $A$ moves in $a$ downward direction and side $B$ in an upward direction. The voltage and current flow is of opposite polarity to that which was induced originally. This action explains the first two important concepts of AC: the current is periodically reversing, and the instantaneous values of the current and voltage are constantly changing.

You have seen that a voltage is induced in a conductor if there is relative motion between the conductor and a magnetic field. A machine in which the magnetic field is provided by a permanent magnet is called a magneto. If the magnetic field is provided by an electromagnet, the machine is called a dynamo. The part of the machine which contains the conductors is called the armature. The motion, which causes the induced voltage, can come by rotating the armature, the field, or both.

A generator is a magneto or dynamo which converts mechanical energy to electrical energy. The mechanical energy is supplied by a motor. A motor is a magneto or dynamo that changes electrical energy to mechanical energy or motion.

A simplified generator is shown in figure 3-4. The armature is a rectangular coil rotating between the poles of an electromagnet. (The motor that drives the armature is omitted from the figure.) $A B$ and $C D$ may be considered to be two conductors at the bottom and top of the coil connected to two rings called slip rings or collector rings. These rings are mounted on and rotate with the armature. They make contact with the outer circuit through brushes $X$ and $Y$. The load on the generator is represented by the resistance $R$.

As the coil rotates about an axis perpendicular to the magnetic field, in figure 3-4, it cuts lines of force and induces a voltage. In the position shown, no voltage is being induced because the conductors $A B$ and $C D$ are moving parallel to the magnetic field, and no lines of force are being cut. As the coil rotates counterclockwise, $A B$ moves up and CD moves down. By the left-hand


Figure 3-4. Simplified AC Generator.

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rule, you can determine that the induced voltage causes electrons to flow from A toward $B$, in $A B$, and from $C$ toward $D$, in CD. Through the outer circuit (R), current flows from $Y$ to $X$ back through the brushes to the conductors. Thus, a complete path is formed for the current with the voltages induced in $A B$ and $C D$ in series. As the coil reaches the point where $A B$ is moving down and $C D$ up, the direction of the electron flow reverses in the conductors and in the outer circuit. Since the currents reverse in conductors $A B$ and $C D$ at the same time, and since the voltage induced in $A B$ is in series with the voltage induced in $C D$, you can consider them as one conductor for purposes of explanation.

Study an end view of this conductor taken at 45 degree intervals as the coil is rotated. Figure 3-5 shows the conductor rotating clockwise in a magnetic field whose flux lines are from left to right. The entire process is repeated every time the conductor makes a complete revolution.


Figure 3-5. End View of a Conductor Rotating Clockwise in a Magnetic Field.
At $A$. The conductor is traveling parallel to the flux lines; no lines are being cut, and the induced voltage is zero.

From A to B. As the conductor travels from A to B, the number of flux lines being cut per second is increasing; the induced voltage is increasing.

From B to C. The induced voltage continues to increase until $C$, the maximum. At this point, the conductor is moving perpendicular to the flux lines and cutting a maximum number per second.

From C to D. The number of flux lines being cut per second gradually decreases until point $D$ is reached. At $D$, the voltage induced is the same as at B.

From D to E. The induced voltage continues to decrease until, at E, it is zero again. At this point, the conductor is again moving parallel to the flux lines.

From E to F. The conductor is now moving upward. The induced voltage is, therefore, in the opposite direction. It increases from zero until, at F, it is equal in magnitude and opposite in polarity to the induced voltage at $B$ or $D$.

From $F$ to G. The induced voltage increases until the maximum, at $G$; this maximum is in a direction opposite to that at $C$.

From G to H. The voltage decreases until, at $H$, it is equal to that at F .
From H to A. The decrease continues until the conductor returns to its starting position, at $A$, where the induced voltage is again zero.

If you make a graph of the voltage induced in the conductor at the various points discussed above, you will get a curve (figure 3-6). Plot the various points in degrees along the horizontal axis and the magnitude of the voltage induced at each point along the vertical axis. Use the horizontal axis as a reference for zero voltage. Consider the voltage induced in one direction as positive and that in the opposite direction as negative.


Figure 3-6. Induced Voltage.
The curve is similar to one developed in mathematics for the variations of the sine function of an angle as the angle is increased. Since this function is called a sine, the curve is known as a sine wave. It can also be used to represent the voltage or current produced by a generator of the type discussed. Because this output periodically changes its direction, it is called alternating current, and the generator is an AC generator. Figure 3-7 illustrates a simple AC generator with its armature in various positions. The sine wave of voltage generated in one revolution of the armature is also shown. Compare armature positions that differ by 180 degrees, and see that the current has reversed itself. The usefulness of representing AC generation this way is discussed next.

## GRAPHIC REPRESENTATION OF AC

In order to represent this action in graphic form, two factors have to be taken into consideration. One, the voltage and current are building up from zero to maximum, first in a positive direction, and then in a negative direction. Thus, voltage or current can be represented by the projections of a rotating vector. Two, it takes time to complete this action. You will find vectors quite useful in the section of this course that deals with the effects of capacitors and inductors on AC circuits. They are used to determine AC impedance's and currents, as well as voltages. Vectors are discussed in detail in the first subcourse of this three-part series.


Figure 3-7. Generation of AC Voltage.
Different positions of the vector are shown in the circles of figure 3-8. Start the vector in the horizontal position, as shown at the top, and rotate it counterclockwise. Each position shown occurs after the vector has rotated 45 degrees from the preceding position. Since it takes time for the vector to rotate, AC voltage can be represented graphically with the horizontal line representing time and the vertical values representing the amounts of voltage, at any time, during the rotation. The length of the vector represents the peak, or maximum, value of the voltage obtained; so, the vertical distance from the end of the vector to the horizontal line will represent the voltage being generated at that instant.

Figure $3-8$ shows the development of the graph of the different values of voltages for each position of the vector. In labeling the time values along the horizontal line, the graph starts at the left with zero time. Each point along the time axis indicates the time elapsed since zero time. For example, if it takes 1 sec to make one complete revolution, the graph can be labeled as shown in figure 3-9. Figure 3-10 combines figures 3-8 and 3-9.


Figure 3-8. Generation of Sine Wave as Represented by a Rotating Vector.


Figure 3-9. One Cycle per Second.


Figure 3-10. A Sine Wave in Time.

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Actually, an ordinary AC generator rotates many times a second so the time values become very small fractions. Generally, the values along the time axis are in terms of the number of degrees completed since the start of the graph. The values on the time axis may be represented in radians as well as degrees (figure 3-11). The term radian is explained in the first subcourse of this three-part series.


Figure 3-11. Relationship of Radians to Degrees.
The radian, like the degree ( ${ }^{\circ}$ ), is a measure of angular distance. Refer to figure $3-11$, as you read the following explanation of the radian. This figure shows a circle having its center at $O$ and a radius, $r$. An arc $A B$, equal in length to $r$, is laid off on the circumference of the circle. The ends of the arc $A B$ are connected to the center of the circle with straight lines $A O$ and $B O$. The angle $\theta$, thus formed by $A O B$, is equal to one radian. The radian is defined as the angle subtended by the arc of a circle equal in length to the radius of the circle. There are $2 \pi$ radians in a circle. In mathematics, $\pi$ is a constant which is equal to 3.14. Understanding the graphing of AC will make the following characteristics and illustrations of AC easier to understand.

## FREQUENCY OF AC VOLTAGE AND CURRENT WAVES

The graph of figure $3-11$ shows the correlation of radians and degrees for one revolution of a circle. Since this same process is repeated many times, one revolution is referred to as a cycle. Half of a cycle is an alternation. In figure 3-11, all values of the wave between 0 degrees and 180 degrees are above the zero reference line; therefore, that portion of the wave is called the positive alternation. The portion between 180 degrees and 360 degrees is the negative alternation.

Recall the graph in figure 3-9. It shows what happens when it takes one sec to complete a cycle. This is referred to as one cycle per second and is expressed with the term hertz, abbreviated Hz . The AC voltage used in house lighting is 60 Hz . The number of cycles per second of an AC voltage or current is its frequency. The time required to complete one cycle is the period ( $P=1 / f$ ). The period of a 60 Hz voltage is $1 / 60 \mathrm{sec}$.

Every AC voltage generated has some frequency. The frequency studied in this course will range from about 16 to 20 Hz , up into the millions of Hz . The frequencies within this spectrum have many uses (figure 3-12). Most electricity is used for lights, motors, heating devices, etc. When electricity is used for power, its frequency is termed power frequency. The most commonly used power frequency is 60 Hz . The lowest frequencies, between 20 Hz and $20,000 \mathrm{~Hz}$, are termed audio frequencies, for these are frequencies of vibration that can be heard. For instance, if an AC current, whose frequency is 400 Hz , flows through a voice coil of a loudspeaker, the loudspeaker diaphragm vibrates accordingly producing an audible tone whose frequency is 400 vibrations per second. However, there is a limit to how high a frequency the human ear can hear. This limit for the normal ear is about $15,000 \mathrm{~Hz}$.


Figure 3-12. Frequency Spectrum.

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Frequencies above the audio range are in the thousands and millions of Hz . For convenience, the expression, a thousand Hz , is shortened to a kilohertz (KHz); twenty thousand Hz, then, would be 20 KHz . Megahertz (MHz) represent millions of Hz ; therefore, 30 MHz means 30 million Hz . The frequency spectrum shown in figure 3-12 is divided into ranges as follows:

```
L-F = low frequencies.
M-F = medium frequencies.
H-F = high frequencies.
VHF = very high frequencies.
UHF = ultra high frequencies.
SHF = super high frequencies.
```

The numbers $1,000 \mathrm{~m}, 100 \mathrm{~m}$, 10 m , etc., in figure $3-12$, refer to the wavelength of the frequencies in meters. For example, one cycle of a voltage which has a frequency of 300 KHz has a length of $1,000 \mathrm{~m}$. The length of one cycle of a wave of 3 x 104 MHz frequency is one centimeter (cm) which is equal to $1 / 100$ of a meter. You will notice that the wavelength becomes shorter as the frequency becomes greater. Compare the wavelength of 3 MHz and 30 MHz . On figure 3-12, the 3 MHz frequency has a wavelength of 100 meters, but the 30 MHz frequency has a wavelength of only 10 m . This is because of the relationship between frequency and period, which was discussed earlier in this lesson.

## ANGULAR MOTION

The linear velocity of a point is the rate at which the point travels along a straight line. The unit of velocity is the distance traveled in a unit of time when the motion of the point is uniform, such as miles per hour (mph), feet per second (fps), and centimeters per second (cps). The same concept is used to measure and define angular velocity.

In figure 3-13, the radius vector (r) is turning about the origin in a counterclockwise direction to generate an angle theta, $\theta$. The angular velocity of such a rotating vector is the rate at which an angle is generated by rotation. When the rotation is uniform, such as in an AC generator, the unit of angular velocity is the angle generated per unit of time. Thus, angular velocity is measured in degrees per second of radians per second.

Radians per second is the more widely used measure of angular velocity. When a rotating vector has completed one revolution, it has gone through an angle of $2 \pi$ radians. The letter $f$ is used as the abbreviation for the frequency of an $A C$ voltage or current. A rotating vector, then, representing AC, completes f revolutions per second. The angular velocity in radians per second is the number of cycles per second (f) times the number of radians in one cycle ( $2 \pi$ ). Two $\pi$ times $f$, therefore, is the angular velocity in radians per second. The angular velocity in radians per second is denoted by the Greek symbol omega ( $\omega$ ). Expressed in a formula, $\omega=2 \pi f$ radians per second. Find the angular velocity of a vector representing 60 Hz AC. Using 3.14 for $\pi$, omega $=2 \pi \mathrm{Xf}=2 \mathrm{X} 3.14$ $\mathrm{X} 60=376.8$ radians per second.

The linear distance covered by a point of object moving in a particular direction is its velocity multiplied by the time it has been moving. Similarly, the


Figure 3-13. Rotating Vector Generating an Angle
angular distance covered by a rotating vector in seconds is its angular velocity $(2 \pi \mathrm{x}$ f) times the time (t). This angular distance is the total angle generated. Then, $\theta=\omega \mathrm{x} t=2 \pi \mathrm{x} \mathrm{f} \mathrm{x} \mathrm{t}$ radians. Find the angle generated by a 60 Hz AC in $0.01 \mathrm{sec}: \theta=2 \pi \mathrm{x} \mathrm{f} \mathrm{x} t=2 \mathrm{x} 3.14 \mathrm{x} 60 \mathrm{x} .01=3.768$ radians. Now, find what the angle is in degrees, $\theta=360^{\circ} \mathrm{x} \mathrm{f} \mathrm{x} \mathrm{t}$ degrees. The angle in the problem is $\theta=360^{\circ} \mathrm{x} 60 \mathrm{x} .01=216^{\circ}$.

## AMPLITUDE

## Determining Amplitude

A graph of the sine values of the different angles, 0 degrees through $360^{\circ}$, using the horizontal axis to represent the angles and the vertical axis to represent the sine values, follows the same pattern as the graph of AC. There is a direct relationship here between the vertical axis and the angle. The vertical projection, $y$, of the end of a rotating vector, $r$, in terms of the generated angle, $\theta$ is $y=r \sin \theta$. (Refer to figures 3-10 and 3-13.) Y is the opposite side of a right triangle where the angle O and hypotenuse (r) are known and the opposite side is computed. The solution of right triangles was covered in the first subcourse of this series, MM0702. This y from any angle is the amplitude of the wave. The amplitude of a sine wave varies as the sine of the angle.

Similar equations can be written about the values of AC current and AC voltage at any instant. The formula for these instantaneous values of voltage is: $e=$ $E_{\max } x \sin \theta$. $E_{\max }$ stands for the maximum voltage induced. To illustrate the above equation, a certain generator induces a peak (maximum) voltage of 100 V . What is the induced voltage at $30^{\circ}$ after the time the voltage is zero? The induced voltage at that instant is: $e=E_{\max } \sin \theta=100 \mathrm{x} \sin 30^{\circ}=100 \mathrm{x} 0.5$ $=50 \mathrm{~V}$.

The formula for the instantaneous value of current is $i=I_{\max } \sin \theta$ $x$ max indicates the maximum value of induced current, and $\theta$ is the number of degrees that the armature wire has turned from the position of zero induction.

Since an AC voltage or current keeps repeating the same cycles, a graph of one cycle is usually sufficient to describe completely an AC sine wave of voltage or current. The vertical distance of any point on the voltage curve for the horizontal axis is the instantaneous value of the voltage. For instance, the voltage at $\pi / 2$ is maximum, at $\pi$ it is zero, and at $3 \pi / 2$ it is maximum in an opposite direction. The graph is convenient for determining voltage at any desired time of the cycle (fig 3-11).

The top of the curve is indicated as the maximum positive value and the bottom of the curve is the maximum negative value. Maximum value often is referred to as peak value. Twice the peak value, or the difference between the maximum positive and the maximum negative, is termed the peak-to-peak value.

## Effective AC Voltage

The AC voltage used for the lights in classrooms is 110 volts. An incandescent light bulb lighted with $110-\mathrm{V}$ AC is just as bright as it is when $110-\mathrm{V}$ DC is applied to the bulb. The $110-\mathrm{V}$ AC is just as effective as $110-\mathrm{V}$ DC. The DC is a steady 110 V , but the $A C$ is constantly changing. The peak voltage must be greater than 110 V for the $A C$ to have the same effect as 110-V DC. Actually, the peak voltage of the $A C$ would be approximately 155 V (110 x 1.41).

The 110 volts is called the effective value of AC voltage. The effective voltage is the value used in ordinary usage of an AC voltage. The 110 volts is actually 110 volts effective value. In a $220-\mathrm{V}$ line, the AC has an effective voltage of 220 V . An AC voltmeter is calibrated in effective values. In every case, the peak voltage is more than the effective. The peak voltage can be found by multiplying the effective voltage by the square root of 2 , or 1.414. To find the peak voltage of 110 V , multiply 110 x 1.414 which gives 155.5 V peak voltage. $\mathrm{E}_{\max }=1.414 \mathrm{x} \mathrm{E}_{\mathrm{eff}}$.

The effective voltage can be determined from the peak by multiplying the peak by 0.707 , which is one divided by the square root of 2 . For instance, a peak voltage of 100 V would be 100 x 0.707 , or 70.7 V effective AC voltage. Figure 3-14 shows the approximate relationship between effective and peak voltage. Note that the effective value is not the same as the average of all the instantaneous values. The effective value of a sine wave of current may be computed to a fair degree of accuracy by taking equally spaced instantaneous values and extracting the square root of their average, or mean, squared values. For this reason, the effective value is often called the root-mean-square (rms) value. $E_{\text {rms }}=0.707 \times \mathrm{E}_{\text {max }}$.

Since the $A C$ or voltage is of sine wave form, the average current or voltage of one cycle is zero, because of the reversal of direction every half-cycle. The term average value actually means the average value of one alternation of a sine wave, such as shown in figure $3-14$, and may be computed fairly accurately by taking the average of many instantaneous values between two consecutive zero points of the curve. Thus, the average value is equal to the average height of any voltage or current peak (loop). The average value of a


Figure 3-14. Instantaneous Values.
sine wave of voltage or current is 0.637 times the peak value. Eavg $=0.637$ $E_{\text {max }}$, and conversely, $E_{\max }=1.57$ Eavg. The relationship between effective, maximum, average, and peak-to-peak values applies to both AC current and AC voltage. All AC current meters and AC voltmeters, unless marked to the contrary, read effective values of current or voltage.

## PHASE

The phase of a voltage or current wave is very important in the analysis of AC circuits because it is used in comparing two or more AC waves. The relative instantaneous amplitudes and polarities of a wave represent the phase of the wave. Two points on a wave are in the same phase if they have the same amplitude and are moving in the same vertical direction, with respect to the zero (reference) point.

The phase of the wave can be expressed in terms of its period. It is the part of the period which has passed since the wave went through the zero position moving in the positive direction. Phase may also be expressed in time; that is, as a fraction of a period. It may also be expressed in angular measure, where one complete cycle represents $360^{\circ}$ or $2 \pi$ radians. The voltage at points 1 and 13 in figure $3-15$ is zero and moving in the positive direction. These two points are in the same phase. The phase of each point is zero. The voltage at point 7 in figure $3-15$ is zero and is moving in the negative direction. Point 7 is $180^{\circ}$ out of phase with points 1 and 13 . Point 7 is $180^{\circ}$ behind point 1 and $180^{\circ}$ ahead of point 13. The phase of point 7 is $180^{\circ}$.

Point 4 on the curve in figure $3-15$ has moved the maximum distance from the reference line in a positive direction. The phase of this point with respect to point 1 is $90^{\circ}$. The phase at point 4 lags that at point 1.

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Figure 3-15. Phases.
Point 5 represents the position of the generator armature ( $30^{\circ}$ ) after it was at the maximum positive position (4). This means that point 5 and point 4 have a phase difference of $30^{\circ}$, or $1 / 12$ of a period or $2 \pi / 12$ radians. The phase at point 5 lags that at point 4. Conversely, the phase at point 4 leads that at point 5 .

Points 4 and 10 on the wave in figure $3-15$ are $180^{\circ}$ out of phase with each other. Point 4 leads point 10. This phase lead can be expressed as $1 / 2$ the period or $\pi$ radians.

Two sine waves generated by two separate generators, which are $180^{\circ}$ out of phase with each other, are shown in figure 3-16. If you compare the phase of each wave at any given instant, you will see that they are opposite each other in phase. For example, point 2 is maximum positive on wave $A$ and maximum negative on wave B. Both waves are zero at points 1, 3, and 5, but notice that the two waves are moving in opposite directions at each of those points. Point 4 is maximum negative on $A$ and maximum positive on $B$. At any given instance, the two waves are moving in exactly opposite directions.

The outputs of two different AC generators are shown in figure 3-17. These two waves are $90^{\circ}$ out of phase with each other. Point 1 is at zero on wave $A$ and maximum positive on wave $B$. At point 2, wave A is maximum positive, but wave $B$ has fallen to zero. At point 3, wave $A$ has fallen to zero, but wave $B$ has gone to maximum negative. Wave $A$ is maximum negative at point 4, but $B$ is zero at this time. Wave $A$ is back to zero at point 5 , but $B$ is now at maximum positive once again. The two waves differ in phase by $90^{\circ}$ or $1 / 4$ of the period or $\pi / 2$ radians. The same phase difference exists at any given time.

The waves shown in figures $3-16$ and $3-17$ have the same frequency. That is why their phase difference remains constant. If waves $A$ and $B$ had different frequencies, their periods would differ and their phase relationship would change from one instant to another. For example, assume that wave A, in figure 3-16, has a frequency twice that of $B$. This means that the period for $A$ is $1 / 2$ the period for $B$. This also means that, for a given period of time,


Figure 3-17. $90^{\circ}$ Out-of-Phase Sine Waves.
there would be two cycles of wave A for one cycle of wave B. You can see that A would go through its maximum positive value twice while B goes through its maximum positive value only once. The same is true regarding the maximum negative values for the two waves.

## COMBINING AC VOLTAGES

Vectors may be used to combine AC voltages of sine waveforms and of the same frequency. The angle between the vectors indicates the time difference between their positive maximum values. The length of the vectors represents either the effective value or the positive maximum value.

The sine wave voltages generated in coils $a$ and $b$ of the simple generators (figure 3-18A) are $90^{\circ}$ out of phase because the coils are located $90^{\circ}$ apart on the two-pole armatures. The armatures are on a common shaft, and when coil a is cutting squarely across the field, coil b is moving parallel to the field and not cutting through it. Thus, the voltage in coil a is maximum when the voltage in coil b is zero. If the frequency is 60 Hz , the time difference between the positive maximum values of these voltages is

$$
\frac{90}{360} \times \frac{1}{60}=0.00416 \mathrm{sec}
$$

Vector $\mathrm{E}_{\mathrm{a}}$ leads vector $\mathrm{E}_{\mathrm{b}}$ by $90^{\circ}$ in the parallelogram in figure $3-18 \mathrm{~B}$ and sine wave a leads sine wave b by $90^{\circ}$ in figure $3-18 \mathrm{C}$. The curves are shown on separate axes to identify them with their respective generators; they are also projected on their instantaneous values. If coils a and b are connected in series and the maximum voltage generated in each coil is 10 V , the total voltage is not 20 V because the two maximum values of voltage do not occur at the same time but are separated one-fourth of a cycle. The voltages cannot be added arithmetically, because they are out of phase. These values, however, can be added vectorially.
$E_{C}$ in figure $3-18 B$ is the vector sum of $E_{a}$ and $E_{b}$ and is the diagonal of the parallelogram, the side of which are $E_{a}$ and $E_{b}$. The effective voltage in each coil is $0.707 \times 10$ or 7.07 V (where 10 V is the maximum), and represents the length of the sides of the parallelogram. The effective voltage of the series combination is $7.07 \mathrm{x} \sqrt{ } 2$ or 10 V , and represents the length of the diagonal of the parallelogram.

Curve c in figure 3-18C represents the sine wave variations of the total voltage $\mathrm{E}_{\mathrm{C}}$, developed in the series circuit connecting coils a and b. Voltage $\mathrm{E}_{\mathrm{a}}$ leads $\mathrm{E}_{\mathrm{b}}$ by $90^{\circ}$. Voltage $\mathrm{E}_{\mathrm{C}}$ lags $\mathrm{E}_{\mathrm{a}}$ by $45^{\circ}$, and leads $\mathrm{E}_{\mathrm{b}}$ by $45^{\circ}$.

Counterclockwise rotation of the vectors is considered positive rotation, thus giving the sense of lead or lag. If $\mathrm{E}_{\mathrm{a}}$ and $\mathrm{E}_{\mathrm{b}}$ (figure 3-18B) are rotated counterclockwise, and their movement observed from fixed position, $\mathrm{E}_{\mathrm{a}}$ will pass this position first and $90^{\circ}$ later, $\mathrm{E}_{\mathrm{b}}$ will pass the position; thus, $\mathrm{E}_{\mathrm{b}}$ lags $\mathrm{E}_{\mathrm{a}}$ by $90^{\circ}$. If the maximum voltage in each coil is 10 V , the maximum value of the combined voltage will be $10 \mathrm{x} \sqrt{2}$ or 14.14 V . See figure $3-18 \mathrm{C}$. The important point to remember is that two out-of-phase voltages may be resolved into a single resultant value by the use of vectors.

Though generated voltages were used to illustrate the point in this case, the same rules apply to voltage drops as well. Any number of out-of-phase voltages may be combined vectorally, as long as they all have the same frequency; that is, as long as they remain a fixed number of degrees apart, like the two generator loops. Only their magnitudes (length of their vectors) may be different.


Figure 3-18. Plotting Combined AC Voltages.

## REVIEW EXERCISES

Circle the letter of the correct answer to each question.

1. What primary advantage was gained by the development of alternating current systems?
a. Eliminated the need for large transformers.
b. Reduced the power losses in transmission.
c. Reduced the frequencies used in transmission.
d. Eliminated the need for large rectifiers.
2. The amount of voltage induced in a coil is NOT dependent on which of these factors?
a. The number of turns in the coil.
b. The strength of the magnetic field.
c. The size of the brushes.
d. The angle at which the coil cuts the lines of force.
3. What conditions are necessary before two AC voltages can have a constant phase relationship?
a. Unequal amplitudes.
b. Equal amplitudes.
c. Unequal frequencies.
d. Equal frequencies.
4. What is the frequency, in Hz , of an $A C$ voltage that has a period of 0.0001 sec?
a. 50,000 .
b. 10,000.
c. 5,000.
d. 1,000.
5. How does the wave length of an AC cycle vary?
a. Directly with amplitude.
b. Inversely with amplitude.
c. Inversely with frequency.
d. Directly with frequency.
6. What formula represents the angular distance of a rotating vector?
a. $\quad \theta=2 f \omega$.
b. $\quad \theta=\omega r$.
c. $\quad \theta=\omega t$.
d. $\quad \theta=2 \mathrm{ft}$.
7. $2 \pi$ radians represent how many degrees?
a. 90 .
b. 180 .
c. 270 .
d. $\quad 360$.
8. What is the most commonly used power frequency?
a. $\quad 240 \mathrm{~Hz}$.
b. 120 Hz .
c. $\quad 112 \mathrm{~Hz}$.
d. $\quad 60 \mathrm{~Hz}$.
9. What symbol expresses the angular velocity in radians per second?
a. $\quad 2 \omega \mathrm{~F}$.
b. $\quad 2 \pi$ F.
c. $2 \omega \pi \mathrm{w}$.
d. $\quad 2 \omega$ P.
10. If a generator armature rotates 60 revolutions per second, how much time in fractions of a second, is necessary to rotate through $\pi / 2$ radians?
a. $1 / 240$.
b. $\quad 1 / 120$.
c. $1 / 80$.
d. $1 / 40$.
11. In a 1 MHz signal, how many complete cycles exist during a 1-sec interval?
a. $1 / 10,000,000$.
b. $1 / 1,000,000$.
c. $10,000,000$.
d. $1,000,000$.
12. What is the average value, in $V$, if the peak voltage is 120 V?
a. 76 .
b. 82 .
c. 88 .
d. $\quad 92$.
13. What is the effective value, in $V$, if the peak voltage is 240 V ?
a. 84.8.
b. $\quad 90$.
c. $\quad 169.6$.
d. 180 .
14. How are two or more out-of-phase AC voltages normally added?
a. Arithmetically.
b. Algebraically.
c. Vectorially.
d. Geometrically.
15. Root-mean-square indicates which voltage value?
a. Average.
b. Maximum.
c. Instantaneous.
d. Effective.
16. If the maximum induced output of a generator is 400 V , what is the output $90^{\circ}$ from the reference point?
a. 0 .
b. 100 .
c. 200 .
d. 400 .
17. What is the period of an AC sine wave?
a. Pulse repetition frequency.
b. Pulse repetition time.
c. Time required to complete one cycle.
d. Time required to complete a half cycle.
18. What unit changes electrical energy to mechanical energy?
a. Motor.
b. Generator.
c. Reactor.
d. Transformer.
19. One AC cycle contains what number of radians?
a. $\quad 63.7$.
b. 57.3 .
c. $\quad 9.42$.
d. 6.28.
20. What is the frequency, in Hz , if the period of an AC voltage is 0.01 sec ?
a. 1 .
b. 10 .
c. 100 .
d. 1,000.

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed five or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

## Lesson 4

## CAPACITANCE AND INDUCTANCE

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to explain what capacitance and inductance are and describe resistive and capacitive circuits and resistive and inductive circuits.

Conditions. You will have this subcourse book and work without supervision.

Standard. You must score at least 70 on the end-of-course examination that covers this lesson and lessons 1, 2, and 3 (answer 27 of the 38 questions correctly).

## CAPACITANCE

In mechanical systems, if an elastic body such as a rubber band is stretched, there is little or no opposition during the first moment of movement. The opposition increases as the rubber band is extended. If the rubber band is stretched and then fixed in position, the work done in stretching it is stored in the band and is returned when the rubber band is released. This mechanical property of elastic bodies illustrates the electrical property called capacitance.

A capacitor is any two conductors separated by an insulating material. When a DC voltage is first applied across a capacitor, considering no losses, the current flow is maximum the first instant and then rapidly decreases to zero when the voltage developed across the capacitor equals the applied voltage. Energy was consumed in building up the voltage. Removal of the applied voltage leaves the capacitor in charged condition, and energy is stored in the capacitor.

The conductors mentioned above are called the plates of the capacitor, and the insulating material is called the dielectric. In figure 4-1A, when the switch(s) is closed, the battery voltage will cause current to flow. The current flow will be maximum the first instant and will decrease as the voltage across the capacitor builds up. This is shown in figure 4-1B. The number of electrons flowing to plate $A$ is equal to the number flowing away from plate B. The capacitor is fully charged when the difference of potential existing across the plates of the capacitor equals the battery voltage. At this time, the current through the capacitor is zero.

After the capacitor is fully charged, the switch may be opened and the capacitor will remain charged since there is no path for the electrons on plate A to reach plate B. The electrostatic field exists between the plates because they are two charged bodies. The electrical energy is stored between the plates. The direction of an electrostatic field has been arbitrarily stated as the direction a unit positive charge would move if placed in that field. The electrostatic field of the charged capacitor of figure 4-1 is shown in figure 4-2.

When a conductor is placed across the plates, the electrons flow from A to B (figure 4-3) returning the capacitor to its uncharged condition. This discharges the capacitor.

A. initialing the voltage flow.

B. DYNAMICS WITHIN THE CAPACITOR.

Figure 4-1. Charging of a Capacitor.


Figure 4-2. Electrostatic Field Within a Capacitor.


Figure 4-3. Discharging of a Capacitor.

## Measure of Capacitance

The ability of a capacitor (C) to store electrical energy is called capacitance. The unit of capacitance is the farad (f), named for Michael Faraday. One f is the amount of capacitance that will store 1 coulomb of charge $\left(6.28 \times 10^{18}\right.$ electrons) when 1 V is applied. The farad is a very large unit, so the microfarad ( $\mu f \mathrm{f}$ ) and picofarad (pfd) are commonly used. One microfarad equals $10^{-6} \mathrm{f}$, and one pfd equals $10^{-12} \mathrm{fd}$. Capacitors in common use range from 1 pfd to several thousand $\mu \mathrm{fd}$.

The relationship between charge (Q), voltage (E), and C may be expressed as:

$$
Q=C E,
$$

where
$\mathrm{Q}=$ charge in coulombs
$\mathrm{C}=$ capacitance in fd, and
$\mathrm{E}=$ voltage in V.

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This relationship shows that an increase in charge (coulombs) will result from an increase in voltage or capacitance.

## Physical Characteristics

Relative Size of Parts. The capacitance of a capacitor is determined by its construction. Capacitance is directly proportional to the area of its plates and inversely proportional to the thickness of the dielectric. This means that, if the plates were made larger, the capacitance would increase. If the plates were moved further apart, the capacitance of the capacitor would decrease.

Dielectric. The dielectric also affects capacitance because the energy is stored in the dielectric. For example, if a certain capacitor using air as the dielectric were immersed in oil, the capacitance would increase to three times as much. We say that oil has a dielectric constant of 3 . Air has a dielectric constant of 1 and is the standard of comparison. The relationship of capacitance, area, thickness of dielectric, and dielectric constant is as follows:

$$
\begin{aligned}
& \mathrm{C}=\frac{\mathrm{Ak}}{\mathrm{t}} \\
& \mathrm{~A}=\text { area of plate } \\
& \mathrm{k}=\text { dielectric constant } \\
& \mathrm{t}=\text { thickness of dielectric }
\end{aligned}
$$

The actual capacitance in pfd is

$$
\mathrm{C}=0.2248 \frac{\mathrm{Ak}}{\mathrm{t}}
$$

where A is in square inches and $t$ is inches. The constant, 0.2248 , in the above equation is necessary to convert from the electrostatic system of measurement (statcoulombs, statvolts, centimeters, etc.) to the more practical system, using such terms as ohms, amperes, coulombs, and inches.

The $k$ of some common insulating materials are given below:

| Material | $k$ | $k$ |  |
| :--- | :--- | :--- | :--- |
| Air | 1. | Material | Lucite |
| Mica | 2.5 to 8. | Quartz | 4. to 3. |
| Bakelite | 5. to 9. | Paper | 2. to 3. |
| Porcelain | 7. |  |  |

The choice and thickness of the dielectric material determines not only the capacitance but also the voltage rating of the capacitor. Since it is usually desirable to have relatively large capacitance in a small space, the dielectric is quite thin. It must have high dielectric strength to avoid puncture by the applied voltage. Capacitors are commonly rated as to capacitance, working voltage, and peak voltage. DC working voltage is the maximum DC voltage the capacitor will stand satisfactorily under certain applied conditions (in its normal temperature range). AC working voltage rating is lower than DC working voltage rating for a given capacitor because the voltage used is the effective line voltage. For example, a capacitor with an AC working voltage rating of

400 V must be able to withstand a peak voltage of 566 V because that is the peak voltage of the 400 VAC. Peak voltage ratings are generally higher than working voltage since most dielectrics can withstand higher voltages for a short time than they can stand continuously.

## Series Connection

You saw earlier that

$$
\mathrm{C}=0.2248 \frac{\mathrm{Ak}}{\mathrm{t}}
$$

where $t$ represents the thickness of the dielectric. If the value of $t$ is doubled, the capacitance would be reduced to one half. Connecting two equal capacitors in series is the equivalent of building one capacitor having a doubled thickness of dielectric. See figure 4-4.


Figure 4-4. Capacitors in Series.
When the upper plate of $C_{1}$ is made negative, the lower plate of $C_{1}$ must become equally positive. The same holds true for $C_{2}$. Since the upper plate of $C_{2}$ is connected to the lower plate of $C_{1}$, these two points are electrically the same and effectively bring the two dielectrics together forming a single capacitor with a doubled thickness of dielectric. In this series circuit, as in all series circuits, the current flow is the same in all parts of the circuit. As a result, the same amount of charge will accumulate on each capacitor, thus the voltage is equally divided between the two capacitors.

Calculation of total series capacitance becomes more difficult when capacitors of unequal value are connected in series. In figure 4-4, two capacitors of equal value are connected in series. By Kirchhoff's law, the sum of the individual voltages must equal the applied voltage. The current is the same in all parts of the circuit. Since the current is the same, the charge deposited on each capacitor will be the same ( $Q=I t$ ). Charge, voltage, and capacitance are related by

$$
\mathrm{Q}=\mathrm{CE} \text { or } \mathrm{E}=\frac{\mathrm{Q}}{\mathrm{C}}
$$

This shows that the voltage on a capacitor is directly proportional to the charge and inversely proportional to the capacitance. The smallest capacitor will have the greatest voltage across it.

Substituting the value Q/C in place of E, in Kirchhoff's law for series circuits $\left(E_{t}=E_{1}+E_{2}\right)$ you get

$$
\frac{\mathrm{Q}}{\mathrm{C}_{\mathrm{t}}}=\frac{\mathrm{Q}}{\mathrm{C}_{1}}+\frac{\mathrm{Q}}{\mathrm{C}_{2}},
$$

Dividing both sides of this equation by $Q$, you get

$$
\frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}
$$

You can use this equation for determining the total series capacitance of any number of capacitors in series.

$$
\frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\cdots \cdots \cdots+\frac{1}{\mathrm{C}_{\mathrm{n}}}
$$

You can rearrange equation as follows:

$$
\begin{aligned}
& \frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}} . \\
& \frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{\mathrm{C}_{2}+\mathrm{C}_{1}}{\mathrm{C}_{1} \mathrm{C}_{2}} \\
& \mathrm{C}_{\mathrm{t}}=\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}
\end{aligned}
$$

This equation is a useful and timesaving device, but may be used only for two capacitors in series.

Capacitors are often connected in series to allow their being used in circuits having a higher voltage than that for which they are rated. For instance, four $500-\mathrm{V}, 4-\mu \mathrm{fd}$ units in series are the equivalent of a single $2,000-\mathrm{V}$, $1-\mu \mathrm{fd}$ unit. Series operation of capacitors of different values should be avoided. Since all units receive the same amount of charge, the voltage is divided in inverse proportion to the capacitance of each unit. One thousand V applied to 1 $\mu \mathrm{fd}$ and $4 \mu f d$ in series will produce 200 V across the larger capacitor and 800 V across the smaller. If both were rated at 500 V , the $1-\mu f d$ capacitor would promptly break down putting the entire $1,000 \mathrm{~V}$ on the $4-\mu \mathrm{fd}$ capacitor, which would also be punctured.

## Parallel Connection

Two equal capacitors in parallel are the equivalent of a single capacitor having doubled plate area and the same dielectric thickness (figure 4-5). The total capacitance, therefore, is the sum of the parallel capacitance, while the voltage applied must be no higher than the rating of the individual units. For any number of parallel capacitors, total capacitance is

$$
\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}+\mathrm{C}_{4}+\ldots \ldots .+\mathrm{C}_{\mathrm{n}} .
$$

## Capacitive Reactance

Since capacitors are used in AC circuits, it is necessary to define the farad in terms of a changing voltage. When a steady voltage is applied to a capacitor, a momentary charging current flows. This current ceases almost immediately.


Figure 4-5. Capacitors in Parallel.
If the voltage were increased or decreased, another momentary flow of current would take place. If the voltage could be constantly changed, a continuous current would flow. A more rapidly changing voltage would cause a larger average current. For a given rate of voltage change, a larger capacitance causes a larger current to flow. The unit of capacitance (fd) may then be defined in terms of a changing voltage.

A capacitor is said to have a capacitance of 1 fd when a change of 1 V per second produces a current of 1 amp. This can be expressed as the equation,

$$
I_{a v}=C \frac{E}{t}
$$

where $I_{a v}$ is the average current that is forced to flow through a capacitance of $C$ fd by a change of $E V$ in $t$ sec.

In figure 4-6, a sine wave has been divided into 4 equal parts along the time base. The time required for one cycle is $1 / f$. The time required for a change of voltage, from zero to $E_{\text {max }}$ is a quarter cycle, or $1 / 4 f$. If a sine wave of voltage with a peak voltage of $E_{\max }$ and $a$ frequency $f$ is applied across a capacitor of $C$ fd, the average current during a quarter cycle is

$$
\mathrm{I}_{\mathrm{av}}=\mathrm{C} \frac{\mathrm{E}_{\max }}{\frac{1}{4 \mathrm{f}}} \text { or } \quad \mathrm{I}_{\mathrm{av}}=4 \mathrm{fCE}_{\max }
$$



Figure 4-6. Relationship Between Time and Frequency.

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In fact, it is true for the entire cycle because each quarter cycle has the same amount of voltage change per second. This equation should be simplified to eliminate having to work with both average and maximum terms in one operation. The relationship between average current and maximum current of a sine wave is then

$$
\mathrm{I}_{\mathrm{av}}=\frac{2}{\pi} \quad \mathrm{I}_{\max } .
$$

You should recognize $2 / \pi$ as the familiar 0.637 . Substituting this value for $I_{a v}$ in its equation,

$$
\begin{aligned}
& \frac{2}{\pi} \mathrm{I}_{\max }=4 \mathrm{fCE}_{\max } \\
& I_{\max }=2 \pi f C E_{\max }
\end{aligned}
$$

Both sides of the above equation are in terms of maximum values and may be changed to effective values, giving

$$
I=2 \pi f C E,
$$

which indicates that the current flowing through a capacitor is proportional to the frequency of the applied sine wave and the capacitance. As in the case of resistive circuits, you find the opposition to the flow of current by dividing the voltage by the current. Then by equation $I=2 \pi f C E$.

$$
\frac{E}{I}=\frac{1}{2 \pi f C} .
$$

where $f$ (frequency) is the Hz and C is capacitance in farads. The angular velocity $2 \pi f$ is often written $w$ (omega), making reactance equal to $1 / \omega$ C. This equation shows that capacitive reactance is inversely proportional to both capacitance and frequency. The manner in which the opposition to current by a capacitor ( $\mathrm{X}_{\mathrm{C}}$ ) changes with frequency is shown in the graph in figure 4-7. As $f$ increases, $X_{C}$ decreases along with the vertical axis. Ohm's Law is applied to capacitive reactance by substituting $X_{C}$ for $R$,

$$
\mathrm{E}=\mathrm{IX}_{\mathrm{c}}, \quad \mathrm{I}=\frac{\mathrm{E}}{\mathrm{X}_{\mathrm{c}}}, \quad \mathrm{X}_{\mathrm{c}}=\frac{\mathrm{E}}{\mathrm{I}} .
$$

The capacitive reactance $\left(X_{C}\right)$ is the opposition offered to the flow of the current by a capacitor. Since the formula for capacitive reactance was found by working with a sinusoidal voltage applied, it follows that the formula

$$
\mathrm{X}_{\mathrm{C}}=\frac{1}{2 \pi \mathrm{fC}}=\frac{1}{2 \pi} \times \frac{1}{\mathrm{fC}}=\frac{0.159}{\mathrm{fC}}
$$

is true only for sine wave voltages.
You can use the two methods discussed above for determining capacitive reactance by applying them to the circuit shown in figure 4-8. If you place AC voltmeters across $R$ and $C$ and insert an AC ammeter in the circuit you get the readings as indicated in the figure.

These values are actual meter readings. Now since you have the circuit current measured, you can calculate the capacitive reactance of $C$ by

$$
\mathrm{X}_{\mathrm{C}}=\frac{\mathrm{E}}{\mathrm{I}}=\frac{80 \mathrm{~V}}{2 \mathrm{amps}}=40 \Omega .
$$

Without knowing the circuit current, you can calculate capacitive reactance with

$$
\mathrm{X}_{\mathrm{C}}=\frac{0.159}{\mathrm{fC}}=\frac{0.159}{1590 \times 2.5 \times 10^{-6}}=\frac{10^{-4}}{2.5 \times 10^{-6}}=\frac{10^{2}}{2.5}=40 \Omega
$$

Notice in figure 4-8 that more voltage is dropped across $R$ and $C$ than is applied from the generator. The introduction of the capacitor in the circuit has produced a strange circuit behavior. This is because the capacitor causes a phase shift between the voltage and current. Therefore you must use vectors to calculate circuit voltage and current. Vectors for series RC circuits will be discussed later.


Figure 4-7. Relationship Between Capacitive Reactance ( $X_{C}$ ) and Frequency ( $f$ ).


Figure 4-8. AC Voltmeters Across an RC Circuit.

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## Phase Relationship in a Pure Capacitive Circuit

The current flowing through a capacitor at any instant depends on the rate at which the voltage across it is changing. Therefore, with a sine wave voltage applied, the current is maximum when the voltage is crossing the zero axis, because the voltage is then climbing or falling most rapidly. When the applied voltage is at its maximum, it is no longer climbing or falling. As it levels off, the current falls to zero. The relationship between current and voltage is shown in figure 4-9.

The voltage and current are seen to be one-quarter cycle, or $90^{\circ}$ apart. Since the positive alternation of current reaches maximum before that of voltage, current leads the voltage. The phase or time relationship between these two sine waves is also shown by vectors in figure 4-9. The general statement can be made that when a sine wave of voltage is applied to a pure capacitance circuit, the current leads the voltage by $90^{\circ}$.

relationship between CURRENT AND VOLTAGE


RELATIONSHIP BETWEEN Sine waves, by vectors

Figure 4-9. Phase Relationship Between Current and Voltage in a Pure Capacitive Circuit.

## Types of Capacitors

The major types of capacitors used in electronics and other electrical devices are, mica, ceramic, impregnated paper, air, and electrolytic.

Mica. The mica capacitor is thin metal plates separated by sheets of mica. The smaller sizes may be only two plates; larger sizes are numbers of plates and mica sheets. Alternate plates are connected together to provide a large plate area. The general arrangement is shown in figure 4-10. For electrodes, some mica capacitors use thin layers of silver deposited directly on the surface of the mica. A mica capacitor with molded bakelite case and wire terminals is shown in figure 4-11. Mica capacitors are made in small capacitances up to about $0.5 \mu \mathrm{fd}$. They can be made for very high voltages, to a high degree of accuracy, and very stable under temperature changes. They are highly suitable for use in high frequency circuits.

Ceramic. Ceramic capacitors have most of the desirable qualities of mica capacitors and, in addition, may be made with a positive or negative temperature coefficient of capacitance. That is, the capacitance may be made to increase or decrease with changes in temperature. Such capacitors are very useful in compensating for temperature changes in nearby circuit components. The ceramic capacitor is made by depositing silver on the surfaces of a ceramic tube (figure 4-12).


Figure 4-10. Mica Capacitor Showing Mica and Metal Sheets.


Figure 4-11. Molded Mica Capacitor (Enlarged).


Figure 4-12. Ceramic Capacitor.

Ceramics are used in transmitters because they meet the exacting requirements of rigid frequency control. They are the only kind of capacitor used for holding oscillator frequencies to the close limits obtained by crystal control. (Such capacitors are usually negative temperature coefficients.) Hermetically sealed ceramic capacitors have been developed for the precision circuitry of electronic instruments. These are hermetically sealed to withstand changes in atmospheric moisture. Also, they will withstand the vibration and shock normally encountered in military electronic equipment. These capacitors permit capacitance tolerances within plus or minus 1 percent and temperature coefficient tolerances within plus or minus 10 parts per million, per degree Celsius.

Impregnated Paper. Probably the most used capacitor has impregnated paper as the dielectric. The paper is saturated with any number of resins, waxes, oils, or synthetic compounds. For conservation of space and for ease of manufacture, these capacitors are made by winding up two strips of metal foil separated by sheets of paper. To reduce the danger of breakdown because of flaws in the paper, several laminated sheets of paper are used, instead of a single, thicker sheet. The units must be carefully sealed to prevent the paper from absorbing moisture. When made for high voltages, the case of the capacitor is often filled with a high-grade mineral oil. Typical paper capacitors are shown in figure 4-13. The paper capacitor has a greater ratio of capacitance to weight than the mica or ceramic type. It is suitable for power and audio frequencies but has excessive losses at higher radio frequencies.

Air. Air is used as the dielectric to make the capacitor variable. Figure 4-14 shows the construction of two kinds. The rotor plates are all fixed to a common movable shaft. The stationary (stator) plates are fastened to fixed terminals. There may be one or two sets of stator plates. Because of the large spacing between plates and the low dielectric constant, air capacitors are rather bulky and are rarely made larger than 500 pfd . They have low losses and are highly suitable for radio frequencies. An advantage is that the dielectric is selfhealing. That means, that after an excessive voltage has been applied, the capacitor can be used again.

Electrolytic. The electrolytic capacitor is important because it provides a very large capacitance in a small space. It is two metal plates separated by an electrolyte. The electrolyte is the negative electrode. The dielectric in an electrolytic capacitor is a very thin oxide film formed on the surface of the positive capacitor plate. This allows a high capacitance in a small space because of the thin dielectric. Electrolytic capacitors can be used in AC or DC circuits, but the construction for each differs. DC capacitors are commonly marked to indicate polarity, which you must carefully observe. A reversal of applied voltage would destroy the dielectric film. AC electrolytic capacitors are usually used only for intermittent operation. A good example is the electric refrigerator's starting motor. Capacitance ratings of electrolytic capacitors are quite high; the smallest readily available size being 8 $\mu \mathrm{fd}$, while larger values run into thousands of microfarads. Voltage ratings are limited to about 450 working $V$. They are not suitable for use in critical circuits, since the capacitance varies greatly with temperature and age. In addition, they have considerable current leakage. Electrolytic capacitors are self-healing if the breakdown current is not too large. A dry electrolytic capacitor is shown in figure 4-15. Cheaper units are also supplied in cardboard containers with wire leads.


Figure 4-13. Impregnated Paper Capacitors.


Figure 4-14. Variable Air Capacitor.


## Figure 4-15. Electrolytic Capacitor.

## INDUCTANCE

The magnetic field windings of a large industrial DC or AC generator are energized with a few amperes of DC at about 220 V . Currents and voltages of this order can usually be interrupted by a switch no larger than the one that turns on the light in your room. Yet, to open the generator field circuit suddenly, even by means of a large and substantially built switch, would probably result in a violent arc of several thousand volts that would melt the switch and destroy the insulation. These results are highly out of proportion to the small amount of energy in the circuit. Where did enough come from to produce such destruction? The answer lies in the ability of a coil to store electrical energy. This property is called inductance.

The following experiments will help you understand inductance. A wire is wound on a cardboard tube as shown in figure 4-16. The ends of the wire are connected to a galvanometer, which is a sensitive measuring device. When a permanent bar magnet is suddenly moved into the coil as shown in figure 4-17, the galvanometer pointer will swing to the right, which indicates that a voltage has been generated and a current forced around the circuit. When the magnet is at rest, there is no indication on the meter. When the bar magnet is suddenly pulled out, the meter needle swings to the left. This time, the direction of the current has been reversed.

The above experiment shows that:

- Relative motion between a coil and a magnetic field produces voltage.
- The polarity of the voltage reverses when the relative motion is reversed.

The direction of current in the coil and the direction of the magnetic field that results from that current, demonstrates Lenz's Law:

- The countervoltage induced in a coil opposes the force which induced it.
- The change of current in a coil sets up a countercurrent opposing this change.


Figure 4-16. Coil of Wire With Current Measuring Device.


Figure 4-17. Inducing Current in a Coil.
From the observations in the previous experiment, you note that the swing of the galvanometer pointer is proportional to the rate at which the magnet is moved. In other words, the voltage induced in the coil is proportional to the rate of change of the flux.

Assuming that the volume of the coil does not change, if the number of turns were doubled the current would increase four times. The induced voltage, which is proportional to the current, must then be proportional to the square of the number of turns, and directly proportional to the rate of change of the field.

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Changing the current through a coil has an important effect upon its operation. Figures 4-18 and 4-19 explain this effect.

When the switch is closed (fig 4-18), current flows and a magnetic field is set up as shown. Since this field is identical to one produced by a bar magnet, it would be reasonable to expect that when this magnetic field is set up, an opposing reaction (Lenz's Law) would occur similar to that observed when the bar magnet was inserted into the coil. That this reaction does occur may be observed when you use an ammeter (with very little inertia), which will respond almost instantaneously to the current flow. Such a meter shows that the current does not jump suddenly to a maximum, but rises at a very definite rate, gradually approaching a final value. Figure 4-19 shows how the current in the coil rises with time. This slow rise in current is caused by self-induction, or countervoltage.

When the switch is closed, the resulting change induces a countervoltage in the coil that opposes the change in current flow. The countervoltage induced is proportional to the rate of change of the current. The greatest rate of current change occurs the instant the switch is closed, since at that instant, the current is changing from zero to some definite value. Also at that instant, the counter electromagnetic force (EMF) is the greatest. As the current levels off to its final value, the field ceases to change and the voltage falls to zero. The current is now limited only by the resistance in the circuit (in spite of the fact that the field is quite strong) because the field is no longer changing.


Figure 4-18. Magnetic Field of a Coil Carrying a Current.


Figure 4-19. Buildup of a Current in a Coil.
It took energy to build up this magnetic field. Most of this energy is stored in the field around the coil and is returned to the circuit when the source voltage is removed. A small amount of the original energy is lost as heat because of the resistance in the coil. When the source of energy is removed, this stored energy acts to keep current flowing in the same direction. If a battery as a source of energy is replaced with a low resistance, the ammeter will not fall immediately to zero but will show that the current takes time to decline, falling quickly at first and finally leveling off to zero. The direction of current flow does not change. This is because a reversed voltage is generated across the coil during the collapse of the field.

The effect of a coil (its inductance) is much the same as mechanical inertia. Just as it takes time and energy to start a flywheel, it takes time and energy to start a current in an inductive circuit. And, just as the flywheel is capable of giving up energy as it coasts to a stop, inductance returns its stored energy as the current decreases. When a flywheel is stopped suddenly, its stored energy produces destructive forces. Similarly, if an inductive circuit is opened suddenly, the rapid collapse of the field produces a large voltage across the coil. This keeps the current flowing, and the stored energy is almost instantaneously converted to heat at the opening switch contacts. If the inductance and the current are large, the contacts may be badly burned or the insulation of the coil damaged.

## Unit of Induction

Since the effects of inductance are due to the voltage induced when the current through it is changed, the unit of inductance can be defined as follows:

A coil of wire has an inductance of one henry if a current change of 1 amp per second will cause a pressure of one $V$ to be set up in the coil.

Inductances are rated in henrys, millihenrys, and microhenrys. A millihenry (mh) equals $10^{-3} \mathrm{~h}$, and a microhenry ( $\mu \mathrm{h}$ ) equals $10^{-6} \mathrm{~h}$.

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## Phase Relationship in a Pure Inductance

Unlike capacitors, which for most purposes contain negligible series resistance, a coil (inductor) contains a considerable amount of resistance. This is due to the resistance of the wire and to eddy currents and hysteresis losses in the core. (Hysteresis is the lagging of the magnetic flux lines behind the magnetizing force.) A commercially available $30-\mathrm{h}$ coil may have from 100 to 500 $\Omega$ of $D C$ resistance. Pure inductance is not considered physically possible. However, when the inductive reactance (opposition of the coil to a change in current) is large compared to the resistance, it is safe to treat a coil as though it were a pure inductance.

With a sine wave of current flowing in an inductance, the countervoltage is greatest, not when the current is greatest, but when the current is changing to the greatest rate. The counter EMF reaches a maximum negative value when the current is increasing most rapidly in a positive direction (figure 4-20).


Figure 4-20. Applied Voltage, Counter EMF, and Current Across an Inductor.
The counter EMF is at all times opposed to the applied voltage that is making the current flow (Lenz's Law). That is, the back voltage is $180^{\circ}$ out of phase with the applied voltage. You can see from the position of the waves along the axis, that the current, in a pure inductance, lags the applied voltage by $90^{\circ}$. The counter EMF is exactly opposite to the applied voltage.

## Inductive Reactance

Because inductance tries to prevent a change in the current flowing through it, a coil will show considerable opposition to the flow of an alternating current. The exact amount of opposition to an alternating current sine wave may
be derived from the fundamental relationship between voltage, inductance, and rate of change of the current. That is,

$$
\mathrm{EL}=\frac{-\mathrm{L} \Delta \mathrm{i}(\text { Change in current })}{\Delta \mathrm{t}(\text { Change in time })},
$$

where the quantities are in basic units; volts, henrys, amps, and seconds.
The time for one cycle of an $A C$ wave is $1 f$ (figure 4-21). The time required for a quarter cycle (which is chosen because the wave from zero to maximum can be examined) is $1 / 4 f$. If a sine wave of current with a peak value of $I_{m a x}$ and a frequency of $f$ is sent through an inductance of $L h$, the average voltage across the coil during a quarter cycle is:

$$
\mathrm{E}_{\mathrm{av}}=\mathrm{L} \frac{\mathrm{I}_{\max }}{\frac{1}{4 \mathrm{f}}}
$$

To simplify the equation into a more usable form, convert

$$
\mathrm{E}_{\mathrm{av}}=\frac{2}{\pi} \mathrm{E}_{\max }
$$

Substituting this value for $\mathrm{E}_{\mathrm{av}}$ in the above equation,

$$
\mathrm{E}_{\max }=4 \mathrm{fL} \mathrm{I}_{\max } \times \frac{\pi}{2}
$$

and

$$
E_{\max }=2 \pi f \mathrm{~L} I_{\max }
$$

Since both sides of the equation are in terms of maximum values, the subscripts may be dropped and written in terms of effective values. Thus,

$$
E=2 \pi f L I
$$

shows that the voltage drop across an inductance is directly proportional to the frequency, inductance, and current.

The opposition to the flow of current is, as with an electrical device, equal to E/I. Rearranging the above equation, we get

$$
\frac{\mathrm{E}}{\mathrm{I}}=2 \pi \mathrm{fL} .
$$



Figure 4-21. One AC Wave Cycle.

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The opposition to the flow of an alternating current by an inductance is called inductive reactance, denoted by XL, and is measured in ohms. The equation then becomes

$$
X_{L}=2 \pi f L,
$$

or in shorter form, letting omega equal $2 \pi$ f,

$$
X_{L}=\omega L
$$

Inductive reactance is directly proportional to inductance and frequency. This is extremely important. It makes it possible to separate DC from AC in the same circuit or low frequencies from high frequencies. When used in this connection, an inductance is often called a choke. How XL varies with frequency is illustrated by the graph in figure 4-22.

Ohm's law is applied to inductive reactance as shown.
$E=I X_{L}$
$I=\frac{E}{X_{L}}$
$X_{L}=\frac{E}{I}$


## Figure 4-22. Variation of XL With Frequency.

## Series and Parallel Connection of Inductors

Rules for connection of inductors (series or parallel) are based on the assumption that there is no coupling between the individual inductors; that is, lines of magnetic force from one do not induce an appreciable voltage in another. Stray coupling is generally negligible between adjacent laminated iron units because the flux is well confined to the core. Air core coils are isolated by spacing them sufficiently or by placing them at right angles to each other.

Inductances connected in series are added similarly to resistances.

$$
L_{t}=L_{1}+L_{2}+L_{3}+\ldots \ldots+L_{n}
$$

Inductances connected in parallel are also combined in the manner of resistances.

$$
\frac{1}{\mathrm{~L}_{\mathrm{t}}}=\frac{1}{\mathrm{~L}_{1}}+\frac{1}{\mathrm{~L}_{2}}+\frac{1}{\mathrm{~L}_{3}}+\cdots \cdots+\frac{1}{\mathrm{~L}_{\mathrm{n}}}
$$

The voltage drop across each series inductor, with a changing voltage or alternating current applied, is directly proportional to $L$ or to XL. With parallel connections, the voltage across each coil is the same, but the current is distributed in inverse proportion of $L$ or XL.

## Factors Determining Size of Inductance

In general, a coil arranged to produce a large amount of magnetic flux will also have a large inductance. The amount of inductance depends on the number of turns, the length, cross-sectional area, and permeability of the magnetic path.

Air core inductance is a term applied to coils having cores of nonmagnetic materials, such as air, wood, or ceramics. The inductance, which seldom exceeds a few hundred millihenrys, is wound in a number of forms. It may consist of a single layer winding on an insulating tube; it may have a large number of layers of wire, or it may be wound with a heavy self-supporting conductor with little or no solid supporting insulation. It is a good idea to reduce the distributed capacitance which results from turns being close to each other. A number of multilayer winding arrangements have been developed that reduce the shunt capacitance considerably.

Iron core inductors are used when large values of inductance are needed. They are made as large as several hundred henrys. Their larger inductance results from a magnetic path of low reluctance, which is a laminated iron core in the form of a closed magnetic circuit. The addition of such an iron core may easily increase the inductance of the coil a thousand times. Iron core inductors are rated for their current-carrying capacity, as well as for their inductance. Typical construction is shown in figure 4-23. Since an iron core coil is generally used in AC or pulsating DC circuits, it must be laminated. With a continually changing flux, a solid core would have voltages induced in it that would cause heavy circulating currents called eddy currents. This would result in heating of the core and losing power. Laminations limit eddy currents to a small value and reduce the power loss.


Typical construction of an iron core coil. The small air gap prevents magnetic saturation of the iron and increases the inductance at high currents.

Figure 4-23. Iron Core Inductor.

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The inductance of an iron core coil is not constant. This is because the permeability of the core (degree to which the core is magnetized) varies as the magnetizing force changes, due to the current flowing through the core. In addition, there is magnetic saturation of the core at high flux densities. A small air gap placed in the core prevents magnetic saturation of the iron and makes it possible to use the coil over a wide range of current flow.

The inductance of an iron core coil, like that of the air core, is dependent on the diameter and the number of turns. In addition, the permeability of the core is a factor. Since the permeability is not constant, the current flow and magnetic current in the circuit must be considered in figuring the inductance.

The laminated core is not suitable for radio frequencies because of eddy current losses. While eddy currents can be reduced by the use of very thin laminations, it is not practical to produce iron sheets thinner than 0.001 in. Laminated iron core coils are useful only at frequencies lower than the radio frequency range. However, by subdividing the iron into a very fine dust, it is possible to produce an iron core suitable for radio frequencies as high as 30 MHz . The iron is suspended in a plastic insulating material, forming a plug that is placed inside the coil. The inductance is readily made variable by adjusting the position of the core.

## SERIES RESISTIVE AND CAPACITIVE CIRCUITS

In a circuit containing only resistance, the voltage across the resistance and the current through the resistance are in phase; in a circuit containing only reactance, the voltage and current are $90^{\circ}$ out of phase. However, some circuits contain both resistance and capacitive reactance (RC).

A series RC circuit is shown in figure $4-24$, where an $A C$ voltage of 10 V is impressed across a combination of $3 \Omega$ of resistance in series with $4 \Omega$ of capacitive reactance. This circuit can be used to illustrate most explanations of a simple series RC circuit.

As in DC circuits, the sum of the voltage drops around the circuit comprising the load must equal the applied voltage. However, in considering resistance and reactance, voltages can no longer be added or subtracted arithmetically.


Figure 4-24. Series RC Circuit.

This is because the voltage drop across the resistance is in phase with the current, but the voltage drop across the capacitive reactance is 90 degrees behind the current.

The combination of capacitance and resistance in the circuit causes the current to lead the applied voltage by some angle less than $90^{\circ}$. Therefore, the current vector is drawn in the first quadrant as shown in figure 4-25. The voltage across the resistor (IR), since it is in phase with the current, is drawn along the same line as the current vector. Because the voltage across the capacitor ( $I_{C}$ ) lags the current by $90^{\circ}$, it is drawn downward at an angle of $90^{\circ}$ with the current and resistance voltage vectors.


Figure 4-25. Current and Voltage Vectors for RC Circuits.
The applied voltage is the vector sum of these two voltages. This vector sum is the diagonal of the rectangle whose sides are ER and EC. Because the diagonal divides the rectangle into two congruent right triangles, then:

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{a}}=\sqrt{\mathrm{E}_{\mathrm{R}}^{2}+\mathrm{E}_{\mathrm{C}}^{2}} \\
& \mathrm{E}_{\mathrm{a}}=\sqrt{(\mathrm{IR})^{2}+\left(\mathrm{IX}_{\mathrm{c}}\right)^{2}} \\
& \mathrm{E}_{\mathrm{a}}=\sqrt{\mathrm{I}^{2} \mathrm{R}^{2}+\mathrm{I}^{2} \mathrm{X}_{\mathrm{c}}^{2}} \\
& \mathrm{E}_{\mathrm{a}}=\mathrm{I} \sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{c}}^{2}} \\
& \frac{\mathrm{E}_{\mathrm{a}}}{\mathrm{I}}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{c}}^{2}}
\end{aligned}
$$

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The quantity $\sqrt{ } \mathrm{R}^{2}+\mathrm{X}_{\mathrm{C}}{ }^{2}$ is the total opposition, the result of having the two oppositions to current flow ( $R$ and $X_{C}$ ) in the same circuit. This total opposition to $A C$ current flow is called the impedance of the circuit and is shown by Z. Since Z is an opposition to current flow, it is measured in ohms.

The impedance in any circuit can always be found if the total voltage and the total current are known. Ohm's law for AC circuits states that the impedance of the circuit is equal to the total voltage divided by the total current. Expressed as a formula, this would be

$$
\mathrm{Z}=\frac{\mathrm{E}_{\mathrm{a}}}{\mathrm{I}} .
$$

By transposing, this formula would be

$$
\mathrm{I}=\frac{\mathrm{E}_{\mathrm{a}}}{\mathrm{Z}} .
$$

or it could be

$$
\mathrm{E}_{\mathrm{a}}=\mathrm{IZ} .
$$

It is also possible to find the impedance if the reactance and resistance of the circuit are known.

$$
\mathrm{Z}=\frac{\mathrm{Ea}}{\mathrm{I}}
$$

and

$$
\frac{\mathrm{E}_{\mathrm{a}}}{\mathrm{I}}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{c}}^{2}}
$$

Then it must follow that,

$$
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{c}}^{2}}
$$

or

$$
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{L}}{ }^{2}}
$$

## Power Dissipation in RC Circuits

Now what about power in the circuit? When current flows through a resistor, power is dissipated in the form of heat. For example, the power used by the resistor can be found by using the formula $P=I^{2} R$.

The capacitor, however, uses no power because a capacitor stores energy rather than uses it. When an alternating force is impressed across a capacitor, power is taken from the source and energy is stored in the capacitor as the voltage increases from zero to a maximum positive value. As the voltage decreases from a maximum positive value to zero, the capacitor discharges and returns this energy to the source. Likewise, during the negative half-cycle, while the voltage varies from zero to a maximum negative value, the capacitor in charging takes energy from the source. This energy is returned to the source as the cycle is completed. During each cycle, energy is transferred back
and forth but none is used. Actually, not quite as much is returned to the source by the capacitor on the discharge as is taken on the charge. The small amount of energy that is not returned to the source is dissipated by the DC resistance of the connection. The DC loss, in comparison with the amount of energy stored, is so small that for all practical purposes the capacitor uses no power.

If a voltmeter is placed in parallel with the capacitor, and a current meter in series with the capacitor, it seems that the capacitor is using power. This power is called reactive power and is given by the relation

$$
P=I^{2} X_{C}
$$

Power is equal to the voltage times the current in a pure resistive circuit. However, in a reactive circuit, this relationship is a false indication and is called apparent power.

$$
\mathrm{P}_{\mathrm{a}}=\mathrm{EI}
$$

The actual power taken by an AC circuit is termed the true power. The formula for it is

$$
P_{\text {true }}=I^{2} R,
$$

or

$$
P=E_{R} I,
$$

where $E_{R}$ is the potential difference across the resistance of the circuit and $R$ is the resistance in the circuit.

Since the apparent power is a false indication, the power factor (P.f.) is the ratio of the true power to the apparent power. That is,

$$
\begin{aligned}
& \text { p.f. }=\frac{P_{\text {true }}}{P_{a}} . \\
& \text { P.f. }=\frac{I^{2} R}{E I}=\frac{I R}{E} .
\end{aligned}
$$

Then

$$
\begin{aligned}
& E=I Z, \\
& \text { p.f. }=\frac{I R}{I Z}, \quad \text { and } \\
& \text { p.f. }=\frac{R}{Z} .
\end{aligned}
$$

Hence, you can get the power factor of a series circuit by dividing the resistance of a circuit by its impedance. Reviewing the impedance vector diagram and the current and voltage diagram, it is true that the trigonometric relationship of $R / Z$ is $\cos \theta$, where $\theta$ is the angle by which the current leads or lags the applied voltage.

Therefore,

$$
\text { p.f. }=\cos \theta .
$$

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In many circuits, it is desirable to find the true power when the applied voltage, the current, and the phase angle are known. The equation for the true power can be derived from the definition of power factor,

$$
P=P_{a} p \cdot f .
$$

Then

$$
P=E I p . f .,
$$

and

$$
P=E I \cos \theta .
$$

To find the true power in an AC circuit, any of the following formulas may be used.

$$
\begin{aligned}
& P=I^{2} R \\
& P=E_{R} I \\
& P=E I \cos \theta
\end{aligned}
$$

## Effect of Frequency Change in RC Circuits

In many AC circuits, the value of the capacitor and the frequency of the applied voltage are given; from this, the capacitive reactance may be found using the formula:

$$
X_{\mathrm{c}}=\frac{1}{2 \pi \mathrm{fC}}
$$

From the above formula, the capacitive reactance is inversely proportional to the frequency. Doubling the frequency of the applied voltage in this problem will reduce the capacitive reactance by one half.

## SERIES RESISTIVE AND INDUCTIVE CIRCUITS

In the discussion of inductance, you learned that the current through an inductance lags the voltage across it by $90^{\circ}$. Also, you learned that the current through a resistance is in phase with the voltage across the resistance. What happens when resistance and inductance are in series in an AC circuit? Such a circuit is shown in figure 4-26, where an alternating EMF is impressed across a combination of resistance in series with inductive reactance.


Figure 4-26. Series RL Circuit.

Although all coils have some resistance in their winding, this resistance is small compared with the reactance of the coil. For this reason, the circuit can be considered just two components, a pure inductance and a pure resistance.

In a series RL circuit, the following phase relationships are always true:

- The current lags the applied voltage.
- The voltage across the resistor lags the applied voltage.
- The voltage across the coil leads the applied voltage.
- The voltage across the coil leads the voltage across the resistor by $90^{\circ}$.

The angles, by which the current and voltage-drops lag or lead the applied voltage, are determined by the size of the coil, the value of the resistor, and the frequency of the applied voltage. Changing any of these three factors will not change any of the above statements, but will change the size of the phase angles and the magnitude of the current and voltage-drops.

## Power Dissipation in RL Circuits

The coil resembles the capacitor as far as power dissipation is concerned. When an alternating current is flowing through an inductor, energy is being taken from the circuit during the quarter cycle that the current is increasing. This energy is stored in the magnetic field. As the current decreases, the magnetic field collapses. The collapsing magnetic field induces a counter EMF, thereby returning the energy to the circuit. This process of storing the energy in the field and then returning it to the circuit happens every half cycle. The overall result is that the energy is transferred back and forth, but none of it is used. Actually, the wire in the coil has some resistance; so, as the current flows back and forth through the coil, some power is dissipated. This power is so small, compared with the energy being stored, for practical purposes, an inductor uses no power.

This apparent use of power is termed "reactive power," just as in the case of capacitors. However, the reactive power of the capacitor is stored in the dielectric field while that of the coil is stored in the magnetic field. The formula for apparent power,

$$
P_{a}=E I,
$$

is applied to any AC circuit, whether it contains capacitive reactance or inductive reactance, or a combination of resistance and reactance. The true power used by an $R L$ circuit is found by the same formula as the true power in the RC circuit.

A summary of phase relationships in AC circuits is in the following paragraphs. A knowledge of these relationships will greatly aid you in understanding the subject of alternating current.

In a series RC circuit--

- The current leads the applied voltage.
- The voltage across the resistor leads the applied voltage.
- The voltage across the capacitor lags the applied voltage.
- The voltage across the capacitor lags the voltage across the resistor by $90^{\circ}$.


## MM0703, Lesson 4

In a series RL circuit--

- The current lags the applied voltage.
- The voltage across the resistor lags the applied voltage.
- The voltage across the coil leads the applied voltage.
- The voltage across the coil leads the voltage across the resistor by $90^{\circ}$.

Effect of Frequency Change in RL Circuits

```
Increasing the frequency in a series RL circuit causes the reactance, the
impedance, and the voltage across the coil to increase; at the same time, the
current in the inductor decreases.
Doubling the inductance would have the same effect as doubling the frequency.
Reducing the resistance one half would also have the same effect on phase
relationship, but the current would be greater.
```


## REVIEW EXERCISES

Circle the letter of the correct answer to each question.

1. What is the current and voltage relationship immediately after the switch is closed in the circuit in figure 4-27, which shows a capacitor charging?
a. Current and voltage are maximum.
b. Current and voltage are minimum.
c. Current is minimum and voltage is maximum.
d. Current is maximum and voltage is minimum.
2. What is the resulting action if switch (S) is opened after the capacitor (C) is fully charged? Se figure 4.27.
a. C loses its charge immediately.
b. C loses its charge gradually.
c. C maintains its charge through the electrostatic field.
d. C maintains its charge through the magnetic field.
3. What occurs when switch $S$ is closed in this circuit where a capacitor is discharging (figure 4-28)? Assume C fully charged.
a. C loses its charge but retains its electrostatic field.
b. C loses its charge but retains its magnetic field.
c. Current and voltage decrease from maximum to zero.
d. Current and voltage increase from zero to maximum.


Figure 4-27. Charging a Capacitor.


Figure 4-28. Discharging a Capacitor.
4. What symbol indicates a capacitor's ability to store electrical energy?
a. C.
b. $\quad X_{C}$.
C. E.
d. $\quad E_{X}$.
5. What formula is used to compute the charge on a capacitor?
a. $\quad Q=C_{e}$.
b. $\quad Q=C E$.
c. $Q=\frac{E}{C}$
d. $\quad \mathrm{Q}=\mathrm{E}_{\mathrm{C}}$.
6. What best describes a PURELY inductive circuit?
a. Counter EMF and the applied voltage are in phase.
b. Current leads the applied voltage by $90^{\circ}$.
c. Current lags the applied voltage by $90^{\circ}$.
d. Counter EMF and the current are in phase.
7. Why are laminated iron cores used in inductors?
a. To reduce the inductance.
b. To reduce flux linkage.
c. To reduce reactance.
d. To reduce power losses.
8. From which formula is the total capacitance of two series-connected capacitors derived?
a. $\quad C_{t}=\frac{C_{1}+C_{2}}{C_{1} C_{2}}$
b. $\quad C_{t}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}$
c. $\quad C_{t}=C_{1} C_{2}$.
d. $\quad C_{t}=C_{1}+C_{2}$.
9. How is the total capacitance determined for any number of seriesconntected capacitors?
a. $\quad C_{t}=C_{1}+C_{2}+C_{3}$.
b. $\quad C_{t}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}$
c. $\quad \frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\frac{1}{\mathrm{C}_{3}}$
d. $\quad C_{t}=\frac{C_{1} C_{2} C_{3}}{\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}}$
10. What is the voltage drop relationship between two unequal capacitors connected in series?
a. The larger capacitor drops the least voltage.
b. The smaller capacitor drops the least voltage.
c. The voltage drops are equal.
d. The voltage drops depend on the frequency.
11. What formula is used to compute the capacitance of parallel-connected capacitors?
a. $\quad \frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1} \times \mathrm{C}_{2} \times \mathrm{C}_{3}}$
b. $\frac{1}{\mathrm{C}_{\mathrm{t}}}=\frac{1}{\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}}$
c. $\quad \mathrm{C}_{\mathrm{t}}=\frac{\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3}}{\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}}$
d. $\quad C_{t}=C_{1}+C_{2}+C_{3}$.
12. What is the reactance, in ohms, of a 20 pfd capacitor at 30 MHz ?
a. 265 .
b. 26.5 .
c. $\quad 2.65$.
d. $\quad 0.265$.
13. The largest voltage will be dropped across which capacitor in the circuit in figure 4-29, which shows capacitors in series.
a. $\quad 8 \mu \mathrm{fd}$.
b. $\quad 5 \mu \mathrm{fd}$.
c. $\quad 3 \mu \mathrm{fd}$.
d. $\quad 2 \mu f d$.


Figure 4-29. Capacitors in Series.
14. What is the impedance, in ohms, of the series circuit in figure 4-30?
a. $\quad 7$.
b. 5 .
c. 3 .
d. $\quad 1$.


Figure 4-30. Series Circuit.
15. What is the phase relationship between current and voltage when a sine wave of voltage is applied to a capacitor?
a. Current lags voltage by $90^{\circ}$.
b. Current lags voltage by $45^{\circ}$.
c. Current leads voltage by $90^{\circ}$.
d. Current leads voltage by $45^{\circ}$.
16. What best describes mica capacitors?
a. Unsuitable for high frequency circuits.
b. Unstable under temperature changes.
c. Unsuitable for high voltage circuits.
d. Stable under temperature changes.
17. What is the most widely used type of capacitor?
a. Ceramic polished.
b. Mica.
c. Air.
d. Impregnated paper.
18. If the values of inductance and frequency are doubled, how is inductive reactance affected?
a. Unchanged.
b. Doubled.
c. Reduced to one half the original value.
d. Increased to four times the original value.
19. What is the power dissipated by the resistor in an RC circuit when 100 W of apparent power is delivered to the circuit, if the power factor is 0.8?
a. 0.008 W .
b. 8 W .
c. 80 W .
d. 100 W .
20. What is a characteristic of an inductor?
a. It opposes a change in voltage.
b. It opposes a change in current.
c. It aids a change in voltage.
d. It aids a change in current.

Recheck your answers to the Review Exercises. When you are satisfied that you have answered every question to the best of your ability, check your answers against the Exercise Solutions. If you missed five or more questions, you should retake the entire lesson, paying particular attention to the areas in which your answers were incorrect.

## END-OF-SUBCOURSE EXAMINATION

Circle the letter of the correct answer to each question. When you have answered all of the questions, use a Number 2 pencil to transfer your answers to the TSC Form 59.

1. What is electricity?
a. Charged matter.
b. Moving charge.
c. Positive charge.
d. Negative charge.
2. What is static electricity?
a. Ionized atoms.
b. Electricity at rest.
c. Charged particles.
d. Attraction and repulsion.
3. What is one important characteristic of magnetic lines of force?
a. Magnetic lines of force are conducted by all materials.
b. Magnetic lines of force are conducted by some materials.
c. Magnetic lines of force move perpendicular to each other.
d. Magnetic lines of force are attracted by air.
4. Which of the following materials makes the best electromagnet?
a. Copper.
b. Stainless steel.
c. Soft iron.
d. Silver.
5. Who of the following discovered and measured the force between two magnets?
a. Maxwell.
b. Gauss.
c. Oersted.
d. Caulomb.
6. What is the definition of the intensity of a magnetic field in which a unit magnetic pole experiences a force of one dyne?
a. Gauss.
b. Oersted.
c. Coulomb.
d. Maxwell.
7. What is the letter representation of the Gauss?
a. B.
b. H .
c. M.
d. F.
8. What is the angular difference between true north and magnetic north?
a. $\quad 71^{\circ}$.
b. $27^{\circ}$.
c. $\quad 17^{\circ}$.
d. $23^{\circ}$.
9. Who discovered that current is directly proportional to the voltage and inversely proportional to the resistance?
a. Kirchoff.
b. Ampere.
c. Voltaire.
d. Ohm.
10. What is the applied voltage in a series circuit composed of a battery, and three resistors whose voltage drops are $E R_{1}=10 \mathrm{~V}, E R_{2}=5 \mathrm{~V}, E R_{3}=15 \mathrm{~V}$. Use Kirchoff's Voltage Law.
a. $\quad 20 \mathrm{~V}$.
b. $\quad 25 \mathrm{~V}$.
c. $\quad 15 \mathrm{~V}$.
d. $\quad 30 \mathrm{~V}$.
11. What is the formula for Kirchoff's current for parallel circuits?
a. $\quad I_{T}=I_{1}=I_{2}=I_{3}$.
b. $\quad I_{T}=E / R$.
c. $\quad I_{T}=I_{1}+I_{2}+I_{3}$.
d. $\quad I_{T}=\sqrt{I_{1} \quad 2+I_{2}}$.
12. What is the unit of electrical power?
a. Watt.
b. Ampere.
c. Ohm.
d. Volt.
13. What is the formula for two or more unlike resistors in parallel?
a. $\quad R_{T}=E / R_{1}$.
b. $\quad R_{T}=R_{1} / N$.
c. $\quad \mathrm{R}_{\mathrm{T}}=\frac{\mathrm{R}_{1} \times \mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}}$.
d.

$$
\mathrm{R}_{\mathrm{T}}=\frac{1}{1 / \mathrm{R}_{1}+1 / \mathrm{R}_{2}+\cdots 1 / \mathrm{R}_{\mathrm{N}}}
$$

14. What current would flow through a 5,000- $\Omega$ resistor that is dissipating 50 W of power?
a. $\quad 100 \mathrm{~mA}$.
b. 101 A.
c. $\quad 10 \mathrm{~mA}$.
d. $\quad 100$ A.
15. What is the formula for total current in a series circuit?
a. $\quad I_{T}=E_{1} / R_{1}+E_{2} / R_{2}+E_{3} / R_{3}$.
b. $\quad I_{T}=I_{1}-I_{2}-I_{3}$.
c. $\quad I_{T}=I_{1}+I_{2}+I_{3}$.
d. $\quad I_{T}=I_{1}=I_{2}=I_{3}$.
16. What is the symbol for alternating current?
a. DC.
b. Hz .
c. F.
d. AC.
17. How did alternating current get its name?
a. From the person who developed it.
b. From one of its characteristics.
c. From the constant current.
d. From the constant voltage.
18. Which of the following statements best describes the current characteristics of an AC system?
a. The electrons move through a circuit in one direction for a period of time, and them move back in the other direction for a like period of time.
b. The electrons move from a point of low potential to a point of high potential.
c. The voltage in an AC circuit never changes polarity.
d. The electrons move through a circuit in one direction for a period of time, and then move back in the other direction for a period of time twice as long.
19. Which of the following is another name for the magnetic lines of force?
a. CEMF.
b. EMF.
c. Flux.
d. Conductor.
20. Which of the following is the machine in which the magnetic field is provided by an electromagnet?
a. Permanent magnet.
b. Dynamo.
c. Magneto.
d. Armature.
21. What is the name of all values of the sinewave between $0^{\circ}$ and $180^{\circ}$ above the zero reference line?
a. One cycle.
b. Positive alternation.
c. Negative alternation.
d. One revolution.
22. Which of the following letter symbols represents ultra high frequencies?
a. $\mathrm{H}-\mathrm{F}$.
b. VHF.
c. SHF.
d. UHF.
23. What happens to the wavelength of an AC cycle if the frequency increases?
a. Increases.
b. Decreases.
c. Remains the same.
d. Doubles.
24. What is the formula used to find the peak voltage of 100 volts effective?
a. Peak voltage $=.707$ x 100 Veff.
b. Peak voltage $=.9 \mathrm{x} 100$ Veff.
c. Peak voltage $=1.414 \mathrm{x} 100$ Veff.
d. None of the above.

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25. Which of the following is another name used when referring to the RMS (Root-Mean-Square) value?
a. Peak value.
b. Effective value.
c. Peak-to-peak value.
d. Average value.
26. What is the effective value of 200 volts peak-to-peak?
a. 70.7 Veff.
b. 141.4 Veff.
c. 14.14 Veff.
d. 707.0 Veff.
27. Which of the following statements best describes capacitance?
a. The opposition the capacitor offers to voltage.
b. The capacitor's ability to store energy.
c. The opposition the capacitor offers to current.
d. The capacitor's ability to store resistance.
28. What factors determine the capacitance of a capacitor?
a. Area of the plates and thickness of the dielectric.
b. Area of the plates and the length of the dielectric.
c. Distance between the plates only.
d. Type of dielectric used only.
29. What is the total capacitance of a circuit with three capacitors connected in series with the following values, $\mathrm{C} 1=.015 \mu \mathrm{fd}, \mathrm{C} 2=.015 \mu \mathrm{fd}$, and C 3 $=.015 \mu \mathrm{fd}$ ?
a. $.5 \mu \mathrm{fd}$.
b. $.05 \mu \mathrm{fd}$.
c. $.015 \mu \mathrm{fd}$.
d. $.005 \mu \mathrm{fd}$.
30. What is the total capacitance of a circuit containing a . $01 \mu \mathrm{fd}$, a . 015 $\mu f d$, and a . $001 \mu \mathrm{fd}$ capacitor, all connected in parallel?
a. $.035 \mu \mathrm{fd}$.
b. $.06 \mu \mathrm{fd}$.
c. $.026 \mu \mathrm{fd}$.
d. $.0026 \mu \mathrm{fd}$.
31. What is the capacitive reactance of a capacitor valued at . $05 \mu \mathrm{fd}$ when a 5 kHz signal is applied?
a. $6363.1 \Omega$.
b. $\quad 636 \Omega$.
c. $\quad 122 \Omega$.
d. $\quad 235 \Omega$.
32. What is the phase relationship of the current and voltage across a capacitor?
a. Current lags the voltage by $90^{\circ}$.
b. They are in phase.
c. Current leads the voltage by $180^{\circ}$.
d. Current leads the voltage by $90^{\circ}$.
33. What is the phase relationship of the current and voltage in a pure inductive circuit?
a. The current leads voltage by $90^{\circ}$.
b. The current lags voltage by $90^{\circ}$.
c. The current leads voltage by $45^{\circ}$.
d. The current lags voltage by $45^{\circ}$.
34. What is the inductive reactance of a circuit containing an inductor valued at 15 mh , with a 10 kHz signal applied?
a. $\quad 942 \Omega$.
b. $\quad 9843 \Omega$.
c. $\quad 98 \Omega$.
d. $\quad 9042 \Omega$.
35. What is the inductance of a circuit containing two 10 mh inductors connected in parallel?
a. 20 mh .
b. $\quad 10 \mathrm{mh}$.
c. $\quad 5 \mathrm{mh}$.
d. $\quad 15 \mathrm{mh}$.
36. What is the total inductance of a circuit containing three 5 mh inductors connected in series?
a. $\quad 5 \mathrm{mh}$.
b. $\quad 15 \mathrm{mh}$.
c. $\quad 1.67 \mathrm{mh}$.
d. $\quad 10 \mathrm{mh}$.
37. What is the applied voltage to a resistive-capacitive circuit if the voltage drop across the resistor is 15 V and the voltage drop across the capacitor is 20 V?
a. $\quad 10 \mathrm{~V}$.
b. $\quad 15 \mathrm{~V}$.
c. $\quad 20 \mathrm{~V}$.
d. $\quad 25 \mathrm{~V}$.
38. What is the total impedance of a circuit when the resistance is $15 \mathrm{~K} \Omega$ and the capacitive reactance is $10 \mathrm{~K} \Omega$ ?
a. $18 \mathrm{~K} \Omega$.
b. $\quad 1.8 \mathrm{~K} \Omega$.
c. $\quad 180 \Omega$.
d. $180 \mathrm{~K} \Omega$.

## EXERCISE SOLUTIONS

## LESSON 1

1. b (see page 3).
2. d (see page 2).
3. c (see page 19).
4. d (see page 1).
5. c (see page 20).
6. a (see page 19).
7. b (see page 16).
8. c (see page 14).
9. b (see page 20).
10. b (see page 14).
11. d (see page 11).
12. c (see page 10).
13. d (see page 10).
14. a (see page 10).
15. c (see page 1).
16. a (see page 19).
17. d (see page 8).
18. a (see page 2).
19. d (see page 14).
20. c (see page 19).

## LESSON 2

1. C (see page 29).
2. d (see page 29).
3. b (see page 29).
4. d (see page 29).
5. a (see page 37).
6. c (see page 33).
7. a (see page 37).
8. C (see page 33).
9. b (see page 33).
10. c (see page 36).
11. b (see page 34).
12. c (see page 38).
13. a (see page 39).
14. c (see page 41).
15. b (see page 41).
16. b (see page 41).
17. b (see page 41).
18. d (See page 41).
19. c (see page 41).
20. a (see page 32).

## LESSON 3

1. b (see page 50).
2. c (see page 52).
3. d (see page 64).
4. b (see page 59).
5. c (see page 60).
6. C (see page 61).
7. d (see page 60).
8. d (see page 59).
9. b (see page 60).
10. a (see page 59).
11. d (see page 59).
12. a (see page 63).
13. c (see page 62).
14. c (see page 66).
15. d (see page 62).
16. d (see page 61).
17. c (see page 59).
18. a (see page 53).
19. d (see page 60).
20. c (see page 59).

## LESSON 4

1. d (see page 72).
2. c (see page 72).
3. c (see page 72).
4. a (see page 73).
5. b (see page 73).
6. c (see page 88).
7. d (see page 91).
8. b (see page 76).
9. c (see page 76).
10. a (see page 76).
11. d (see page 76).
12. a (see page 78).
13. d (see page 79).
14. b (see page 94).
15. c (see page 80).
16. d (see page 80).
17. d (see page 82).
18. d (see page 89).
19. c (see page 96).
20. b (see page 84).
